AN ANALYTICAL APPROACH TO THE THREE-DIMENSIONAL STELLAR(SOLAR) WIND SOLUTIONS WITH MAGNETIC FIELD

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1. Introduction

Although more than two dacades have past since Parker[1] predicted the existence of the solar wind, no satisfactory solution to the combined problem of stellar-wind and magnetospheric structures has been obtained yet. The main difficulties seem to be arisen from the following two points.

Firstly, the inclusion of the magnetic field $\mathbb{B}$ and the stellar rotation Ω breaks symmetry. Though in the absence of them the spherically symmetric, therefore one-dimensional, treatment is possible, in the presence of them the three-dimensional treatment is in general required. This difficulty can be reduced to some extent by considering only axisymmetric magnetic fields around a rotation axis, as most researchers do. In our treatment, however, this point is overcome by adopting a coordinate-independent description (i.e. the vector representation) in which the expressions of physical quantities are the same for both aligned- and oblique-rotation cases.

Secondly and this is the most troublesome, the outer boundary conditions can not be known explicitly until the problem is solved. Therefore we use here a kind of successive-approximation method starting from a well known situation. For simplicity, we assume that the stellar magnetization has only a dipole moment μ and that all time variations are due only to the stellar rotation (the quasi-steady condition). The modifications of this dipole magnetic field are investigated in turn which are respectively due to the effects of 1) stellar rotation in the asymptotic regions (the solutions in this approximation are called the asymptotic solutions), 2) stellar rotation in the whole vacuum space (the vacuum solution), 3) corotating plasma around a star (the corotating-plasma solution), 4) inertia of a rotating plasma (the centrifugal-wind solution) and 5) pressure and gravity forces (the stellar wind solution in general).

We have already solved the problems up to the third step. The fourth step is now in progress.

2. Asymptotic Solutions[2]

The expression is well known for the vector potential $\mathbb{A}_0$ and the magnetic field $\mathbb{B}_0$ produced by a dipole moment μ in a vacuum, which is at rest in an inertial frame. In the first step of the successive approximations, the effects of stellar rotation is considered in two asymptotic regions, the near zone ($r \ll r_L$) and the wave zone ($r \gg r_L$), where r is the radial distance from the center of a star and $r_L = c/\Omega$ is the light radius.

In the near zone, the vector and scalar potentials are obtained from a general relativistic consideration, and it is shown that the Backus term[3] in the scalar potential results from the transformation law of the potentials between the rest and rotating frames.

In the wave zone, the dipole approximation is used.

The results thus obtained coinside with that of Deutsch[4] as can be confirmed by writing out all the components in the spherical polar coordinate.

3. Exact Vacuum-Solution[5]

The effects of stellar rotation are exactly included in the solution by neglecting all sources (the electric charge and current densities) except for the stellar magnetization current. In this approximation the scalar potential can be put equal to zero. All the effects of rotation are expressed in terms of the operators acting on the zero-order solution $\mathbb{A}_0$: they are the rigid-rotation, retardation and radiation operators. This solution reduces in the asymptotic regions to the asymptotic solutions obtained in the previous step, and is the generalization to a general oblique-rotation case of the solutions obtained by some previous authors[6] in the orthogonal-rotation case ($\mu \perp \Omega$).

The vacuum solution is used[7] to infer the structure of the neutral sheet (or the polarity-reversal-surface) in the heliomagnetosphere. This is a three-dimensional counter part of the solar sector boundaries.

4. Corotating-Plasma Solution[8]

In this step of approximation, a corotating plasma sphere of radius b ($b \leqq r_L$) around the star is taken into account. The plasma is assumed to corotate strictly. In reality, since the mass of the plasma is non-zero the centrifugal force drives an outward plasma flow even if other non-electromagnetic forces are negligible. The inertial force, which is approximately given in terms of the gradient of the non-Backus part of the scalar potential, is considered to be balanced by an imaginary force. Thus, instead of solving a dynamical equilibrium with the plasma outflow, we solve only Maxwell's equations for corotating plasma region ($r \leq b$) and for the outer vacuum space ($r > b$).

The obtained structure of magnetic lines of force is not so drastically different from that in the vacuum case, even in the limit of $b \to r_L$.

5. Centrifugal-Wind Solution

Inclusion of the inertial terms in our consideration yields a formula for the electric field (generalized Ohm's law) such as

$$\mathbb{E} + \frac{1}{c}\mathbb{v} \times \mathbb{B} = \frac{1}{en}(q\mathbb{E} + \frac{1}{c}\mathbb{j} \times \mathbb{B})$$

in a perfectly conducting plasma, where $\mathbb{E}$ and $\mathbb{B}$ are the electric and magnetic fields, $\mathbb{v}$ and n are the velocity and density of the plasma, q and $\mathbb{j}$ are the electric charge and current densities, respectively, and e is the charge unit. The right-hand side of this equation can be rewritten in terms of the inertial terms by using the equation of motion. Generally speaking, the appearence of this term implies the "violation of flux freezing".

Our preliminary analyses[9)] strongly suggest the coexistence of closed magnetic lines of force and plasma outflow across them, as in the numerical results of Kuo-Petravic et al.[10)].

References

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