THEORETICAL PREDICTION OF EIGENFREQUENCIES OF THE SOLAR FIVE MINUTE OSCILLATION

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Abstract

Problems of calculating theoretical eigenfrequencies of the solar five minute oscillation are discussed. First, the numerical accuracy of calculations and uncertainty in physics, which might influence theoretical eigenfrequencies, were examined. It was found that the precision of our numerical calculations was good enough to compare meaningfully theory with observations. Then, the dependence of theoretical eigenfrequencies of five minute oscillation modes on the envelope structure of the sun was investigated. It was found that the p mode frequencies are quite strongly correlated with the depth of the convection zone. Comparison of theory with observation suggests that the solar convection zone is about 200,000 km deep.

1. Introduction

Recent observations by Deubner (1975, 1977) and by Rhodes, Ulrich, and Simon (1977) together with parallel theoretical development by Ando and Osaki (1975), and by Ulrich and Rhodes (1977) have now established beyond doubt that the solar five minute oscillation is identified as the global eigenmodes of the sun. This discovery has opened a new field of research called "solar seismology" in which observed oscillations may be used to probe the solar envelope structure (Gough 1977, Ulrich and Rhodes 1977). To achieve this objective, one must study the sensitivity of the computed eigenfrequencies to variations in both the basic solar model and the assumptions of the normal mode theory. In this work, we examine this problem, and results of theoretical eigenfrequencies are then compared with the most recent observations by Deubner, Ulrich, and Rhodes (1979) in the diagnostic (k,ω)-diagram.

2. The Equilibrium Models

The equilibrium models were constructed by combining the Harvard Smithsonian

Reference Atmosphere (HSRA) with envelope models calculated by using Vitense's mixing length theory for convection. The mixing length of convection, ℓ, was taken to be proportional to the local pressure scale height, H_p, and its ratio $\alpha=\ell/H_p$ was varied to obtain several equilibrium models with different depths of the convection zone. We have constructed models by integrating inwards from a point in the chromosphere to a depth of about 5×10^5 km.

One of equilibrium models was chosen as a standard because it was thought to match a model of the interior of the sun. It was used for various accuracy tests of eigenfrequencies. Its model parameters were: the hydrogen abundance X=0.745, the heavy element abundance Z=0.02, the mixing length ratio $\alpha=\ell/H_p=1.75$, and the depth of the convection zone $=1.95\times10^5$ km.

3. Method of Computations

Eigenfrequencies of oscillations were calculated mostly by solving the equations of linear adiabatic nonradial oscillations. The boundary conditions used were that the vertical displacement vanished at the base of the envelope and that the Lagrangian pressure perturbation vanished at the upper surface. The governing differential equations were integrated by second-order accuracy centred finite differences using the programme described by Baker et al. (1971), in much the same was as had been done by Ando and Osaki (1975, 1977). Subsequent improvements to the eigenfrequencies were made by substituting into variational integrals, which were evaluated to fourth-order accuracy.

4. Accuracy of Normal Model Analysis

Test were made for numerical accuracy by varying the number of mesh intervals N and using Richardson extrapolation to estimate the limiting eigenfrequencies as $N\to\infty$. It was found that eigenfrequencies calculated with 200 mesh intervals differed in most cases less than 0.1 % from those with $N\to\infty$. Comparison was also made between eigenfrequencies calculated by directly solving difference equations and those calculated from variational integrals. Their difference was again found to be small.

Having examined numerical accuracy, we have investigated sensitivity of eigenfrequencies upon several physical assumptions used. We computed nonadiabatic eigenfrequencies by treating radiative transfer in the Eddington approximation, and compared them with adiabatic frequencies. Eigenfrequencies of normal modes were also computed subject to various other upper boundary conditions: vanishing of Eulerian pressure perturbation and of vertical displacement, and matching to a running wave in an isothermal corona. The results depended upon the modes selected, but in all cases the deviations from the reference f and p mode adiabatic eigenfrequencies were less than the uncertainties in the corresponding observations.

5. Sensitivity of Eigenfrequencies on the Equilibrium Models

This analysis was performed by computing the changes in the eigenfrequencies of modes of degree (of spherical harmonics) $\ell=200$ and 600 resulting from variations in the parameters determining the equilibrium model. Composition, the atmospheric T-τ relation, the mixing length were varied separately. Also, models were computed using a nonlocal theory.

Broadly speaking, the eigenfrequencies were most strongly influenced by the mixing length, which determines the adiabat deep in the convection zone and hence controls the depth of that zone. A model having larger α and hence deeper convection zone generally yields a lower frequency for a given p-mode, and its eigenfrequencies tend to fit better with observed ridges of five-minute oscillation in the (k,ω)-diagram. Eigenfrequencies calculated for the standard model with $\alpha=\ell/H_p=1.75$ and the convection zone depth of 195,000 km fit fairly well with observations, but they are still slightly higher than observations indicated. The results of calculations are illustrated in figure 1 for one of equilibrium models (with $\alpha=\ell/H_p=2.5$) that reproduced observations tolerably. This model had a convection zone 230,000 km deep. Figure 1 shows the frequencies of its lowest order eigenmodes, regarded as continuous functions of the horizontal wavenumber, superposed on the power spectrum of Deubner et al. (1979).

Changes in the atmospheric structure had very little effect on the f and p mode eigenfrequencies, except for the chromospheric modes. Thus we conclude that provided the eigenvalue analysis is a good representation of the five minute oscillations, the observations imply that the depth of the solar convection zone is at least about 2×10^5 km.

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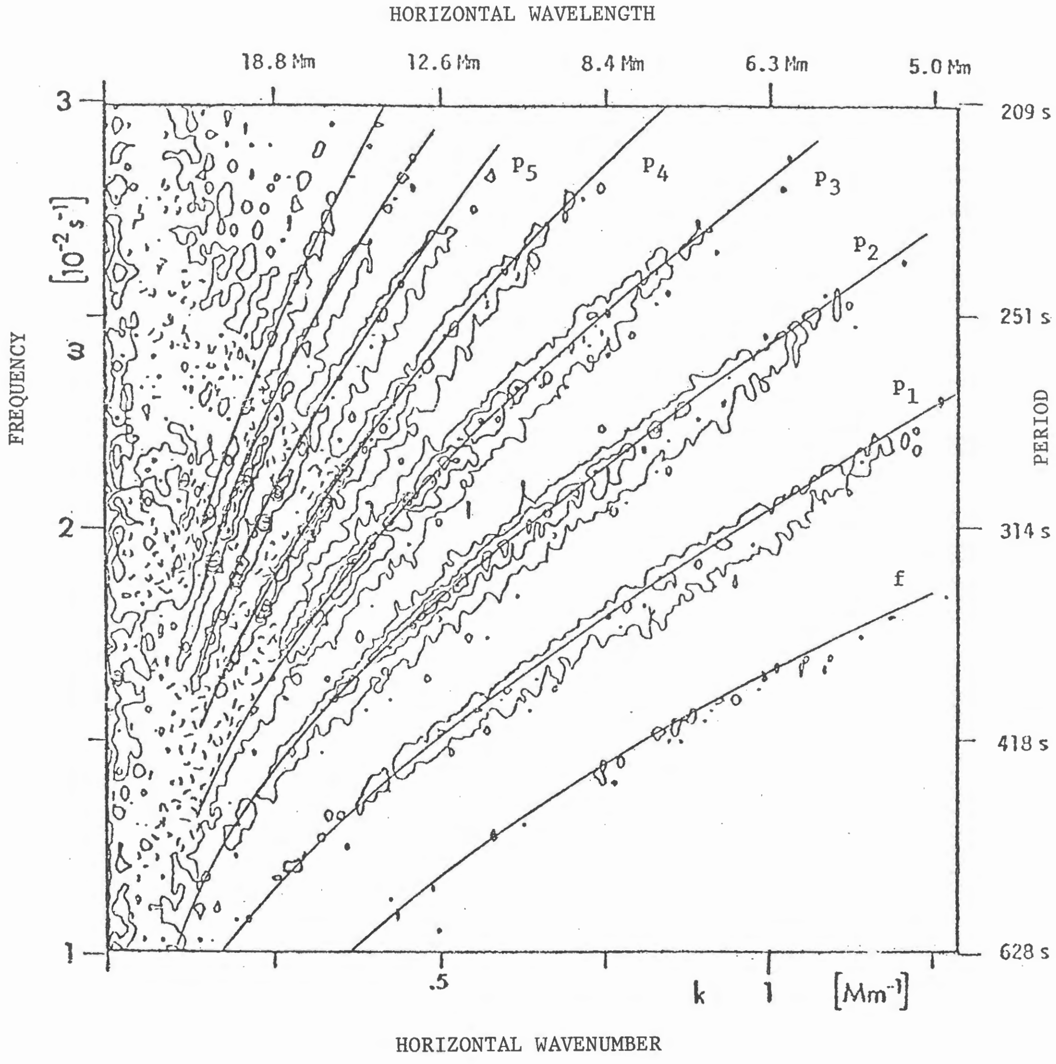

Figure 1. Eigenfrequencies of a model envelope, regarded as a continuous function of the horizontal wavenumber k, superposed on the power spectrum obtained by Deubner et al. (1979).