Stability of Lundquist Field and Prominence Exposing Helical-Like Patterns

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Abstract

The stability of Lundquist field is analysed in the light of energy principle.The results show:(1)for disturbances of $m = 0$ mode,Lundquist-field is stable; (2) for disturbances of $m = 1$ mode, Lundquist-field will turn instable on certain condition. Whether the field will keep stable or not depends on parameters as the helical pinch angle,radius and length of the prominence. Comparison between theory and observations reveals that kink-instability in Lundquist field may be a critical physical reason for eruptive prominences exposing helical-like patterns.

1. Introduction

A helical prominence gives much information on its physical characteriztics , especially its electromagnetic properties.Many authors have studied the regularity and properties of its evolution and eruption either observationally or theoretically (Vrsnak et al. 1988; Vrsnak 1988; Srivastava 1991; Xu 1992; Wu et al. 1993) .A series of important results has been obtained. Xu(1992) and Wu et al.(1993) proposed particularly that Lundquist field may be a good configuration to discribe the inner magnetic distribution of the helical prominence by analyzing both some quiescent and eruptive prominences of such kind.In this paper,we begin with discussing the stability of Lundquist field , analyze it according to energy principle and give a stable criterion. Comparing theory with observational data of 28 helical prominences, we are not only confirmed in the reliability of using Lundquist field to describe a helical prominence , but also glad to find out the physical factor leading to its eruption .

2.Analysis of stability of Lundquist field

The linear symmertrical force-free Lundquist field in the cylindrical system takes the following form:

$$B_r = 0, B_\theta = B_1 J_1(\alpha r), B_z = B_1 J_0(\alpha r), \tag{1}$$

where B_1 is the magnetic field at the axis, α is the constant force-free factor, $J_0(\alpha r)$ and $J_1(\alpha r)$ are the zero and first order Bessel functions respectively.

According to the MHD theory of stability, the energy principle with which to judge the stability of a magnetic configuration is: when $\delta w > 0$,the magnetic configuration is stable; otherwise, unstable.In the following section ,the $m = 0$ and $m \neq 0$ modes are discussed respectively. When $m = 0$, δw is determined by Xu and Tang(1987):

$$\delta w = \frac{\pi}{2\mu_0}\int_0^{r_0} dr[rB_z^2(\frac{d\xi_r}{dr})^2 + (\frac{B_z^2}{r} + 2\mu_0\frac{dP}{dr})\xi_r^2] + \frac{\pi}{2\mu_0}\frac{k^2}{}\int_0^{r_0} B_z^2\xi_r^2 r dr \tag{2}$$

Given the force-free property of Lundquist field,$dp/dr = 0$,each term in the integral function in equation(2) only takes the positive value , i.e.,$\delta w > 0$ .Thus,for $m = 0$ perturbances ,Lundquist field is stable. For $m \neq 0$ mode,it's shown that kink instability of $m = 1$ mode occurs quite easily compared with $m > 1$ mode. Therefore, we only need to discuss the instability of $m = 1$ mode .For this mode ,δw takes the form:

$$\delta w = \frac{\pi}{2\mu_0}\int_0^{r_0} dr\{\frac{r(krB_z + B_\varphi)^2}{k^2r^2+1}(\frac{d\xi_r}{dr})^2 + [\frac{(krB_z - B_\varphi)^2}{r(k^2r^2+1)} - \frac{2B_\varphi}{r}\frac{d}{dr}(rB\varphi) \tag{3}$$

$$-\frac{d}{dr}(\frac{k^2r^2B_z^2 - B_\varphi^2}{k^2r^2+1})]\xi_r^2\} + \frac{\pi}{2\mu_0}\int_0^{r_0} dr\frac{(krB_z+B_\varphi)^2}{r}\xi_r^2$$

For Lundquist field , given the property of Bessel function, it can be proved that:

$$\delta w = \frac{\pi}{2\mu_0}\int_0^{r_0} drF(r)\xi_r^2 \tag{4}$$

$$F(r) = k^2r[\frac{2(k^2r^2B_z^2 - B_\varphi^2) + (krB_z + B_\varphi)^2(k^2r^2+1)}{(k^2r^2+1)^2} \tag{5}$$

It can be seen that longwave disturbances stimulate kink-instability easily. For plasma cylinder of a prominence with length L, the largest disturbed wavelength is $\lambda_{max} = L$.So,there is $k_{min} = 2\pi/\lambda_{max} = 2\pi/L$.For plasma cylinder of a prominence, usually there is $2\pi r/L < 1$. So $(kr)_{min}^2 \ll 1$. For long-wave disturbances , $k^2r^2 + 1 \approx 1$. As the basic characteriztics of helical prominences may be described in figure 1,the pinch angle θ can be represented in term of:

$$\tan\theta = \frac{B_\varphi}{B_z} \tag{6}$$

Substituting equation(6) into $F(r)$,we get

$$F(r) = k^2 r B_z^2 (3k^2r^2 + 2krtg\theta - tg^2\theta) \tag{7}$$

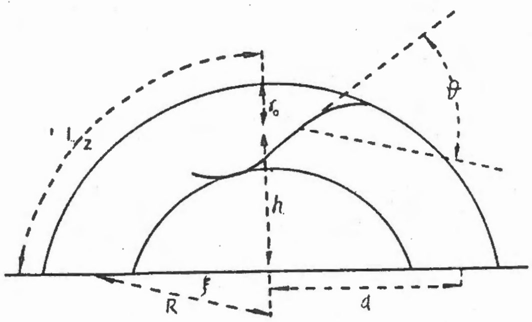

Fig.1.The Schematic of Helical Prominences

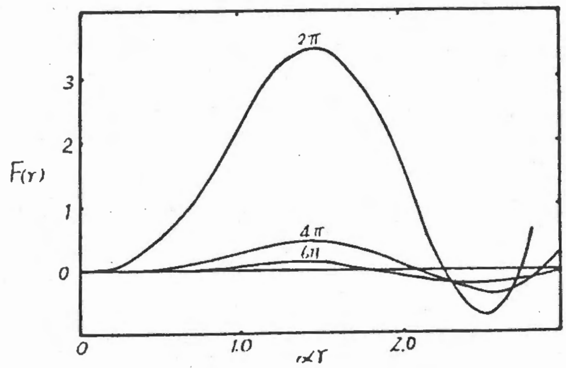

Fig.2.The Curve of F(r)

By looking into the property of $F(r)$, we can tell whether δw is positive or negative With αr as the varible. αL as a parameter. $F(r)$ curve is obtained (figure 2) , where αr_c expresses the value at the point where the curve intersects across the x- axis, i. e. $F(r_c) = 0$.The features of Bessel function imply more than one zero point, among which, the one with the smallest r_c (the first non-zero point intersecting across x-axis) is the most interesting to us. In the following discussion,r_c stands for the smallest value of r_c.When $r < r_c$,$F(r) > 0$; When $r > r_c$,positivity of $F(r)$ cannot be determined.Therefore,if the radius of a prominence $r_0 < r_c$,the integral function of equation (4) is always positive, i. e. $\delta w > 0$ and the prominence is stable. If $r_0 > r_c$, $F(r)$ can be either positive or negative and δw is possibly negative.In this case,the instability may occur and the prominence may take on an eruptive state. Other properties of $F(r)$ illustrated on figure 2 are also noteworthy: (1) r_c increases as αL decreases;(2)when αL is relatively small, the integral function contributes to the sum more positively than otherwise. So,for $\alpha L < 6\pi$, there must be $\delta w > 0$;while for $\alpha L > 6\pi$.there appears a different case,in which positivity or negtivity of δw will be determined by r_0.

3.Comparison with observational data

To make the comparison, we chart an $\alpha L - \alpha r_c$ curve from figure 2(figure 3) . Aaccording to the property of $F(r)$seen from figure 2,the $\alpha L - \alpha r_c$ plane may be devided into two large regions(see figure 3(a)) : in the shaddowed region Lundquist field is stable;and in the right-up region ,unstable . Vrsnak et al(1991) give basic observed data of 28 helical prominences (15 quiescent ones and 13 eruptive ones)and their typical parameters including z,r_0/d (confused to R in original table) and θ,where $z = h/d$.The physical meaning of h,d is seen in figure 1, from which ,it can be deduced that:

$$L = R(\pi \pm 2\zeta) = d(\frac{1+z^2}{2z})[\pi \pm 2\cos^{-1}\frac{2z}{1+z^2}] \tag{8}$$

thus,

$$\alpha L = \frac{\alpha r_0}{r_0/d}(\frac{1+z^2}{2z})[\pi \pm 2\cos^{-1}\frac{2z}{1+z^2}] \tag{9}$$

With the value of θ in table 2 (Vrsnak et al 1991) and $tg\theta = J_1(\alpha r)/J_0(\alpha r)$,$\alpha r_0$ may be calculated. Therefore,using equation(8)and the data in table 2, one can work out αL of the prominence.It should be pointed out that Vrsnak et al. classified 13 eruptive prominences into 3-types: the ones of the first are just beginning to erupt (ON) ; those of the second are undergoing fierce eruption(E) ; and the rest are on a stage of post acceleration of eruptive prominence(PE).Furthermore ,groups of observed data are available,including z, r_0/d and θ on different moments,and we have got up to 49 groups for all the 28 prominences. As an example, there are at hand 10 groups of data for the prominence on August 18, 1980. This facillitates our comparison of the theory with observations.

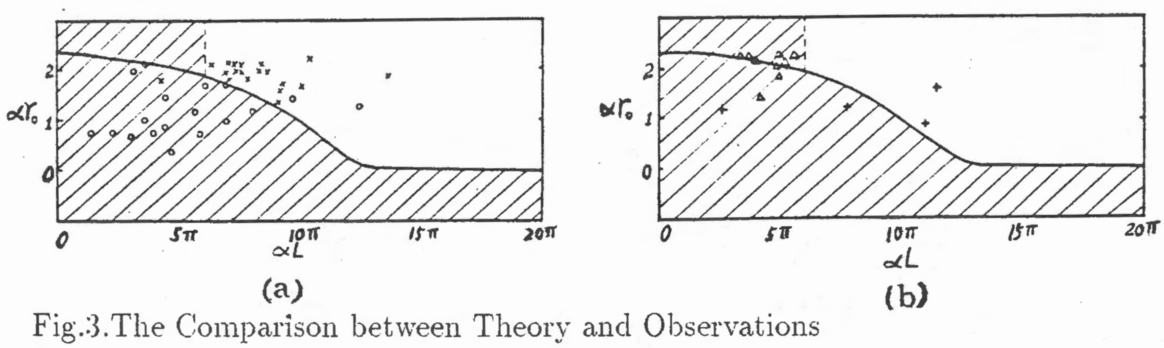

Fig.3.The Comparison between Theory and Observations

According to different groups of αL and αr_0, the quiescent prominences can be located in $\alpha L - \alpha r$ plane.Figure 3(a)shows that,except for two points,the other 1 6 observed points fall into the stable region. One of the exceptional points stands very near the $\alpha L - \alpha r_c$ curve with another point (No. 15,on March 15,197 7, on the following moment) belonging to the same prominence located in the stable region. Thus, the prominence on March 15, 1977 may be considered as a quiescent one approching to a stable state in $\alpha L - \alpha r$ plane. In another word, the prominence on October 2, 1970 is the only exception. In the same way, 18 groups of data of eruptive prominences find their positions in unstable region in $\alpha L - \alpha r$ plane, seen from figure 3(a) .The results provide reliable evidence that kink-instability of Lundquist field induces prominence into eruptive phase.

For further comparison, we have plotted another two types of eruptive prominences (ON, PE) in $\alpha L - \alpha r$ plane(figure 3(b)).The result shows that their properties are not as apparent as those of the two types just discussed. We'd rather ascribe it to observational limitations.As for some prominences defined at the onset of eruptive phase, for lack of observable data of their relative progressing, it is difficult to tell to what extent the data concide with theory. For those on post acceleration phase, as a result of magnetic energy- releasing by kink-instsability, the inner Lundquist field relaxes from instability to stability. So, generally speaking, they should fall in stable region; while in the case of the prominences far away from the end of eruptive phase, Lundquist field stays on unstable state with the corresponding points located in unstable region in $\alpha L - \alpha r$ plane. To sum up , in spite of observational limitations, we are still sure that theory is not contradictory with observations.

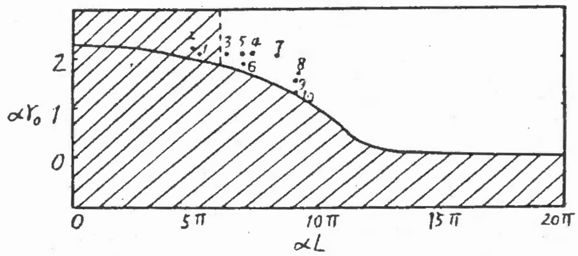

Fig.4. The Evolution of the Eruptive Prominence on August 18, 1980

Figure 4 shows that the parameters of Lundquist field in the eruptive prominence on August 18,1980 vary with time in $\alpha L - \alpha r$ plane.The whole process is seen clearly that the prominence began at stable state(point 1-2) , then went instable (eruptive points from 3 to 9) and returned to stable state in the end (point 10).This provides an eloquent argument that Lundquist field induced the prominence into erupting.

Through the comparison between theory and observations given by figure 3 and figure 4, it is fully proved that whether a helical prominence will keep quiescent or turn eruptive depends chiefly upon the existence of stability in Lundquist field.

4.Conclusion and Discussion

Through the analysis of instability of Lundquist Field and the comparison between theory and high qualitive observational data of helical prominences ,it is shown that:

(1) Inside the prominences, at least those exposing helical patterns, Lundquist field is the basic configuration;

(2) Lundquist field is stable for $m = 0$ mode.On some conditions,the long-wave disturbances of $m = 1$ mode may induce kink-instability to occur in helical prominences, which makes an important physical reason for eruptions;

(3) The pinch angle θ, radius r_0 and length L of the prominence are the basic parameters to determine whether the helical prominence will take on stable state or not.

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