GUAS Asian Solar Physics Winter School
Lecture 3

Introduction to helioseismology

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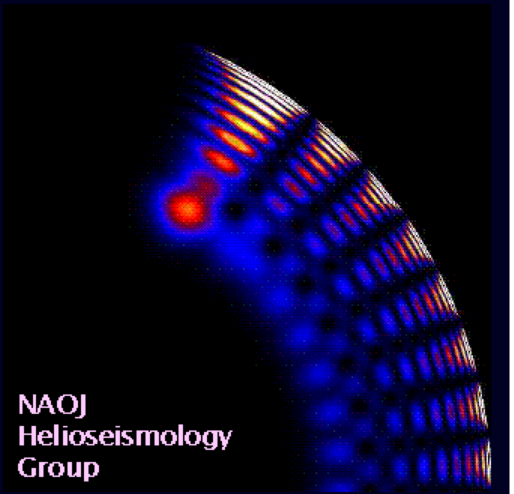


Today’s lecture

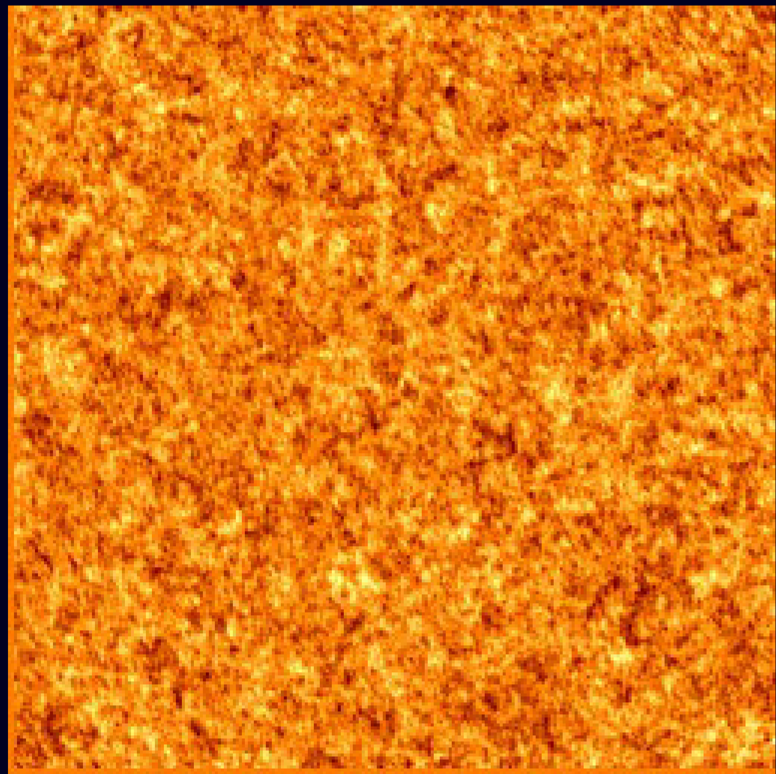
- The sun is oscillating
- How we can see through the photosphere of the sun
- What are the issues with the sun anyway?
- Inversion of Global-mode frequencies and main results
- Development of Local helioseismology

The 5-minute oscillations

- Leighton et al (1962) looked at the solar surface velocity field $v(x,y,t)$
- They found an oscillating component
  - The sun is a variable star!
  - The period is about 5 minutes
- How can we measure the velocity field?
  - The line-of-sight component of solar surface velocity can be measured by Doppler measurement

Dopplergram movies

- From SOHO/MDI

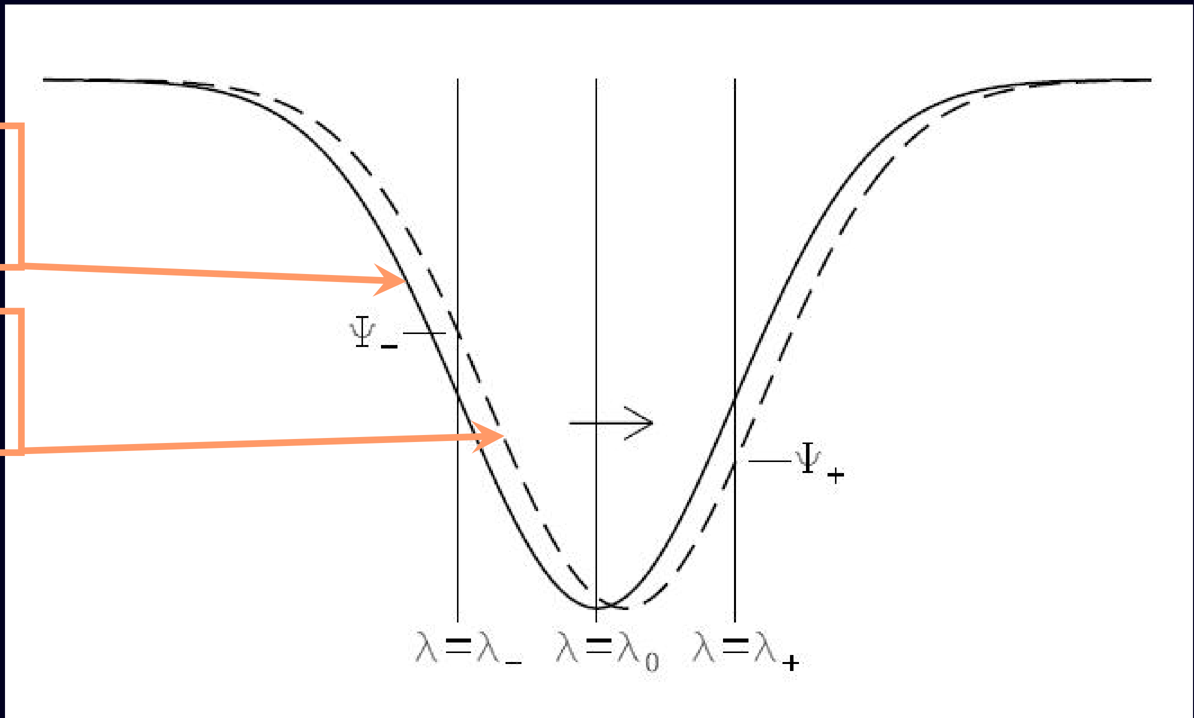


Doppler velocity measurement

- A schematic view of how an absorption line undergoes Doppler shift

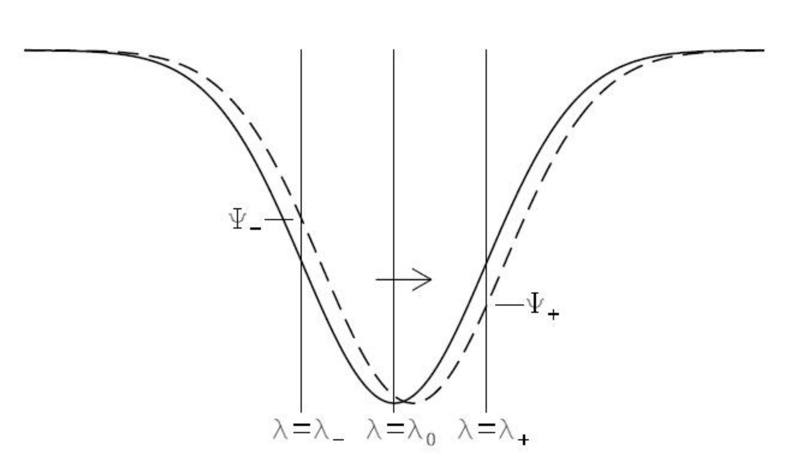
Surface element stationary

Surface element in motion



Doppler velocity measurement

- The difference in the profiles gives us the Doppler velocity



ψ_0 : the line profile

λ_0 : the line centre

$\lambda_\pm = \lambda_0 \pm \Delta\lambda$

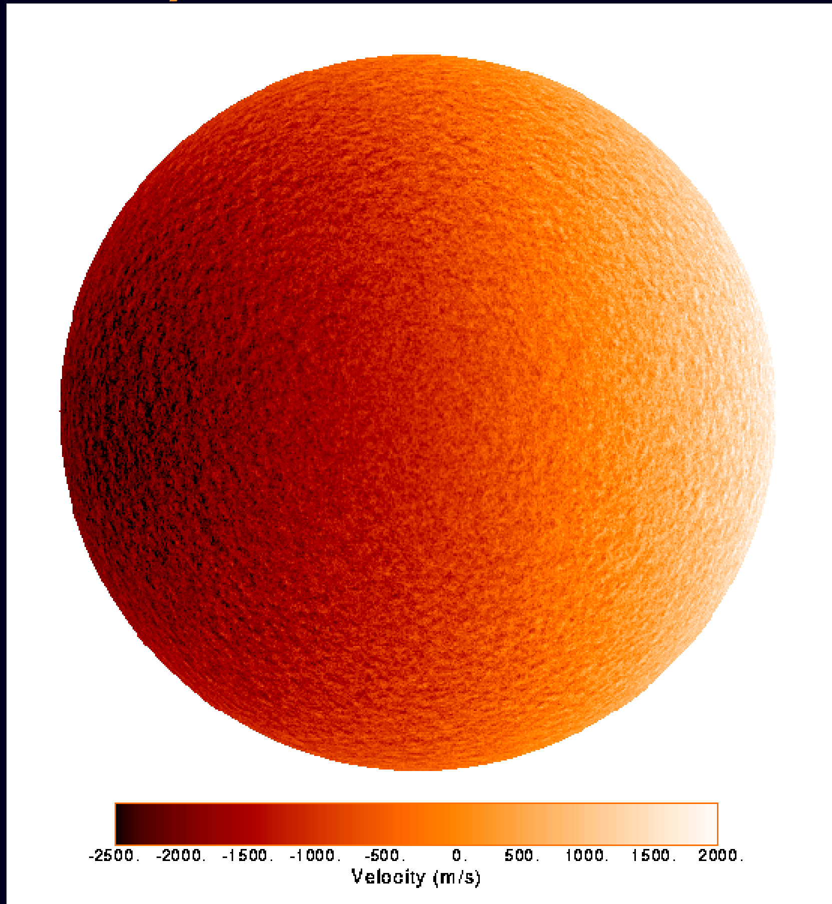
$$\psi_+ - \psi_- = \psi_0(\lambda + \Delta\lambda - \lambda v/c) - \psi_0(\lambda - \Delta\lambda - \lambda v/c) \propto v/c$$

Dopplergram

- Dopplergram obtained by SOHO/MDI

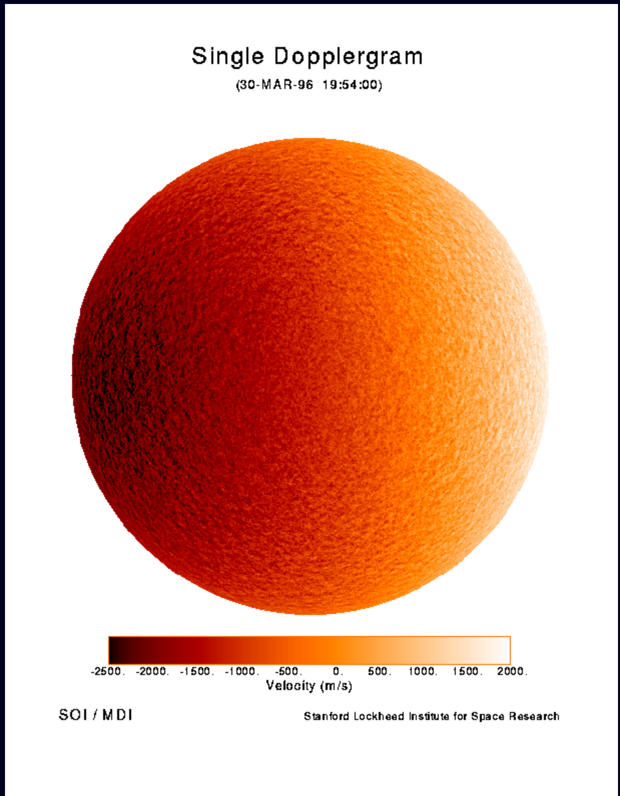
$-2\text{km/s} < v < 2\text{km/s}$

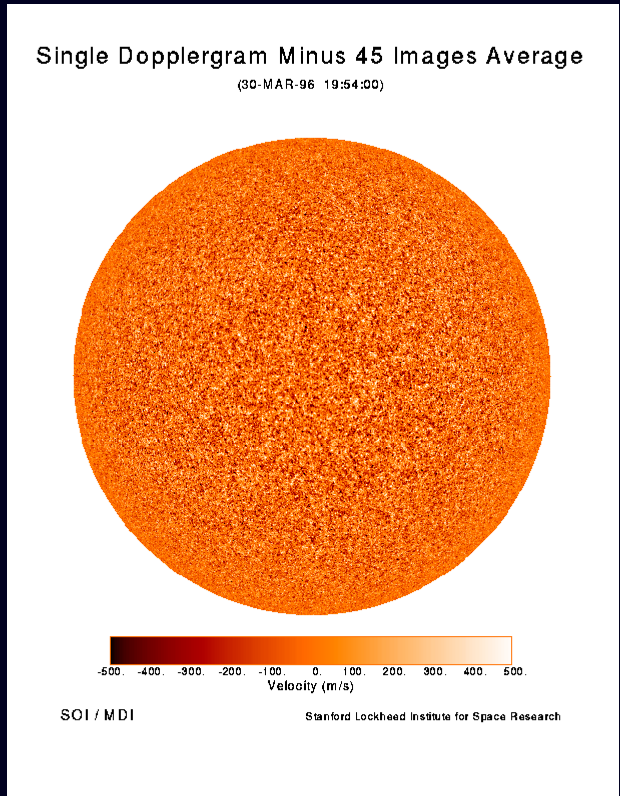
Dominated by solar differential rotation



Dopplergram

- By subtracting 45-min average we can filter out rotation and granulation

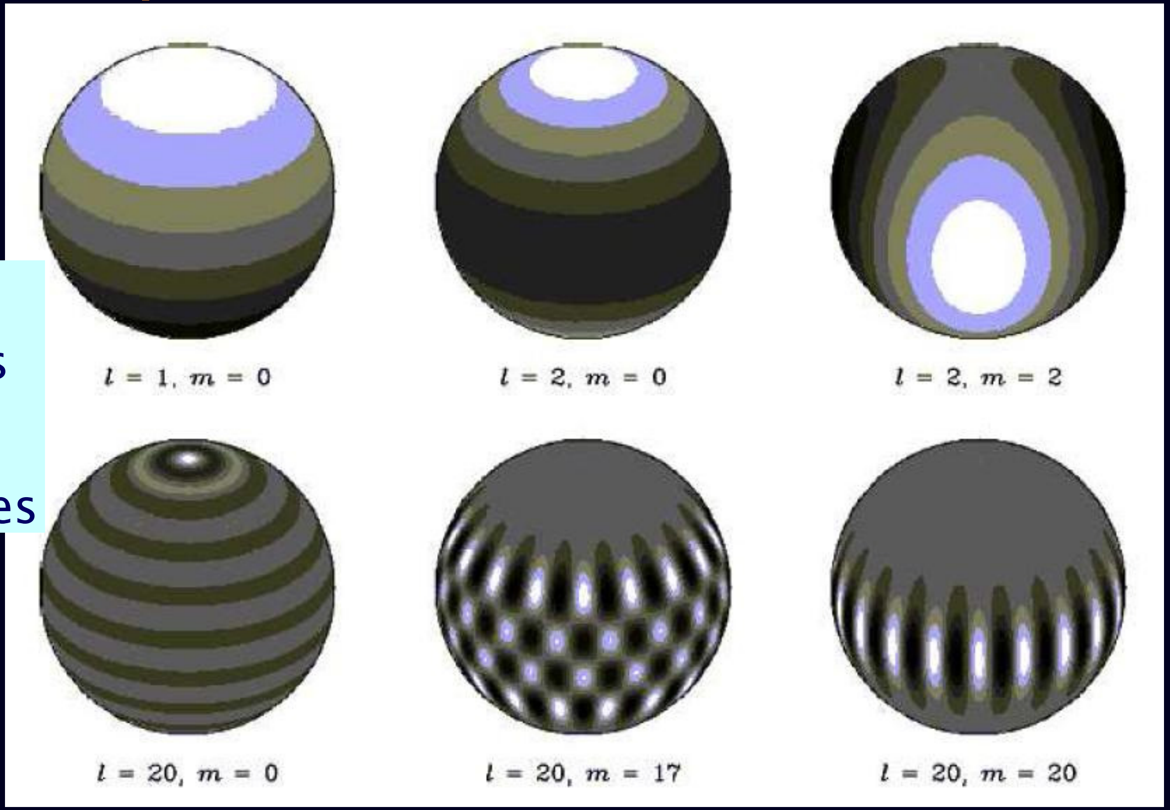




FSH decomposition

- Any scalar function on sphere can be expanded in spherical harmonics

l : the number of nodal lines
m : the number of azimuthal nodal lines



FSH decomposition

- Any scalar function on sphere can be expanded in spherical harmonics

$$f(\theta,\phi)=\sum_{lm} f_{lm} Y_l^m(\theta,\phi)$$

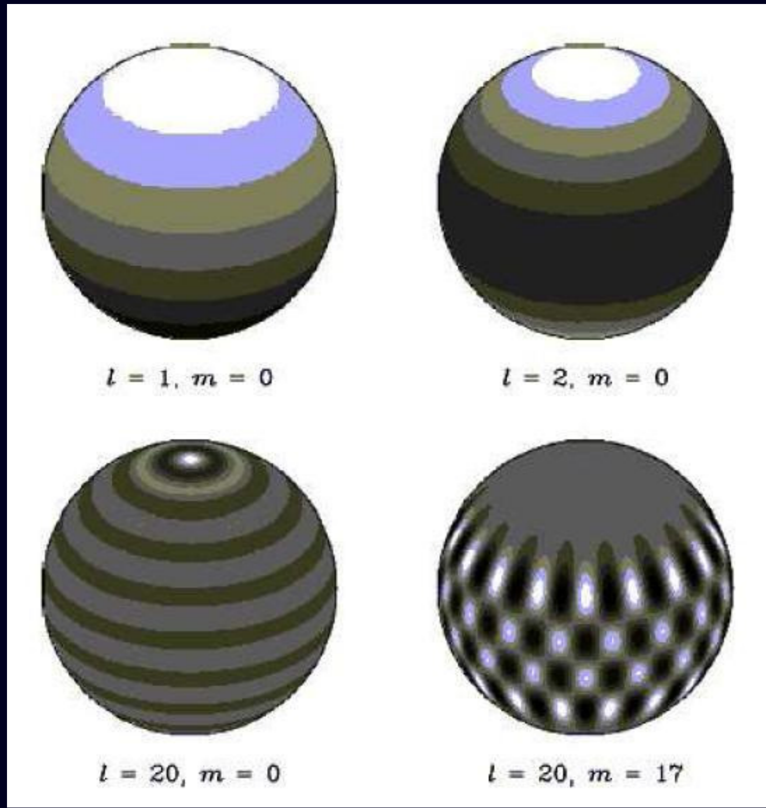
$$Y_l^m(\theta,\phi)=P_l^m(\cos\theta)e^{im\phi}$$

l : degree

m : azimuthal order

$f(\theta,\phi)$: symmetric

$\Rightarrow f_{lm}$ indep't of m



FSH decomposition

- For simplicity, we assume we observe the radial velocity (rather than the line-of-sight velocity)
- In spatial domain, the velocity field can be expanded in spherical harmonics

$$v(\theta,\phi,t) = \sum_{lm} A_{lm}(t) Y_l^m(\theta,\phi)$$

$v(\theta,\phi,t)$: radial velocity field

$Y_l^m(\theta,\phi)$: spherical harmonic function with degree l and azimuthal order m

FSH decomposition

- In time domain, Fourier decomposition comes in handy

$$A_{lm}(t) = \int a_{lm}(\omega) e^{i\omega t} d\omega$$

- Then we have Fourier-Spherical-Harmonic decomposition of the velocity field

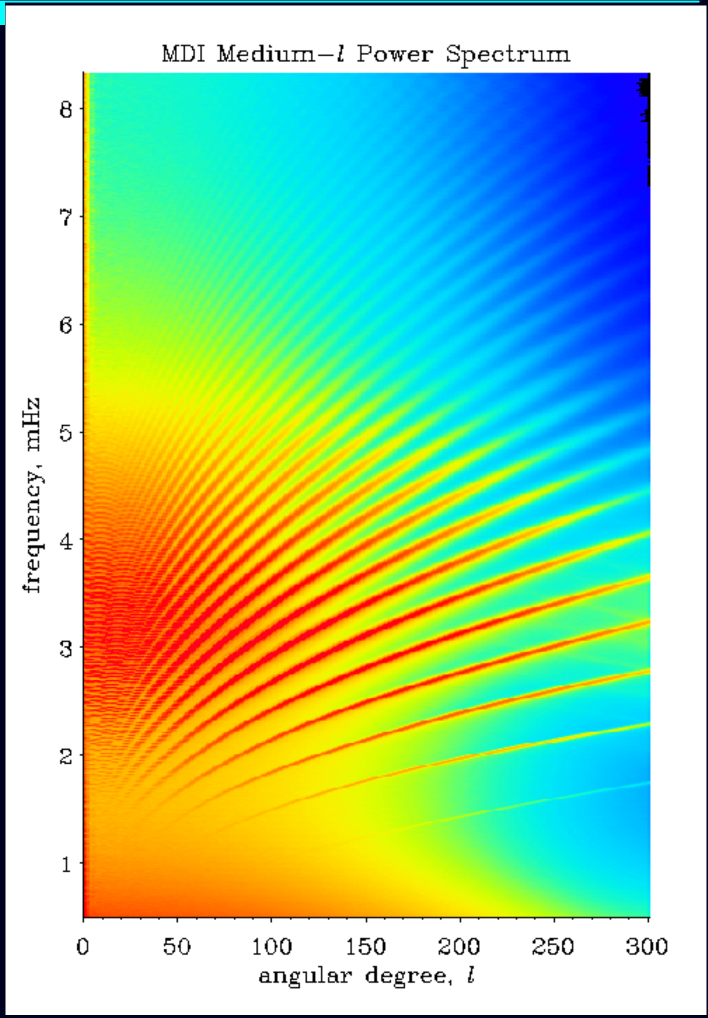
$$v(\theta,\phi,t) = \sum_{lm} \int d\omega\, a_{lm}(\omega) Y_l^m(\theta,\phi) e^{i\omega t}$$

$$a_{lm}(\omega) = \frac{1}{2\pi} \int d\Omega dt\; v(\theta,\phi,t) Y_l^{m*}(\theta,\phi) e^{-i\omega t}$$

The k–ω diagram

- The power spectrum

$$p_l(\omega) = \frac{1}{2l+1} \sum_m | a_{lm}(\omega) |^2$$



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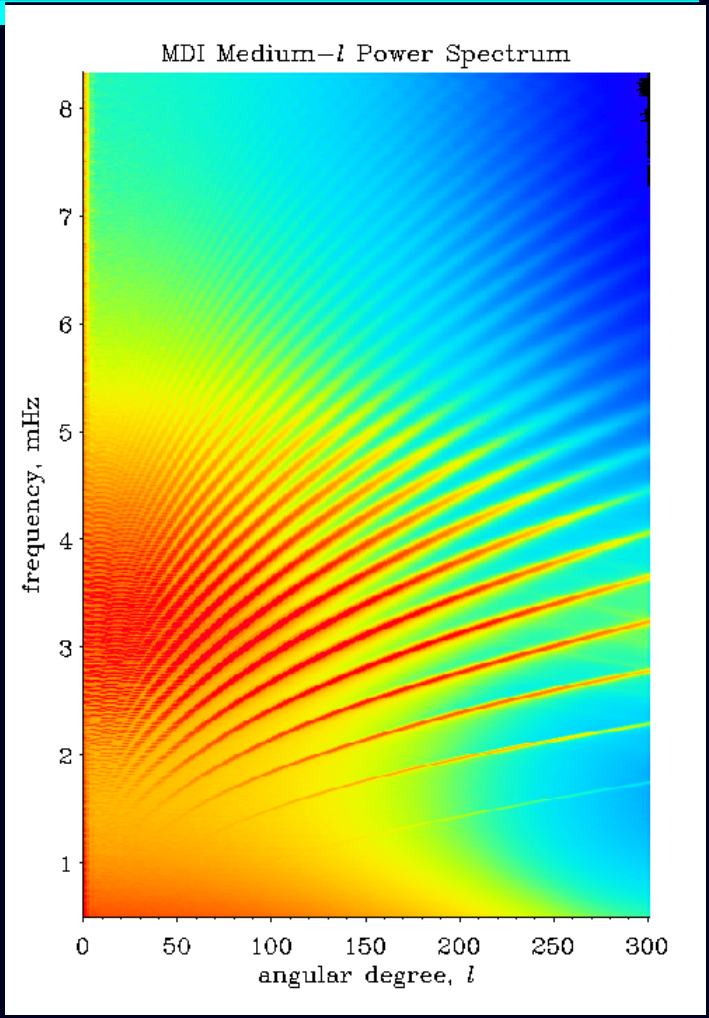
The k–ω diagram

- The power spectum

$$p_l(\omega) = \frac{1}{2l+1}\sum_m |a_{lm}(\omega)|^2$$

- The characteristic 'ridge' structure
  - A full explanation would be too lengthy, but it is a signature of acoustic eigenoscillations

p–mode oscillations

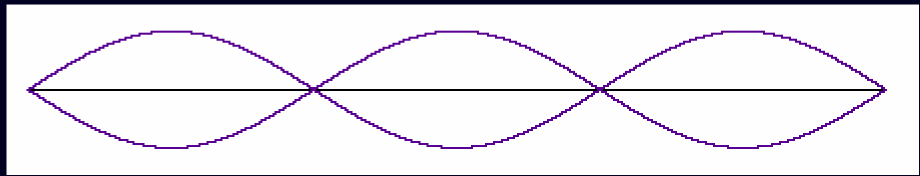


Eigenoscillations

- We have an eigenoscillation mode when waves that travel inside a medium interfere with *itself* constructively
- This happened only when a certain condition on frequency is met
- What this condition is depend on various properties of the medium such as wave propagation speed & geometry

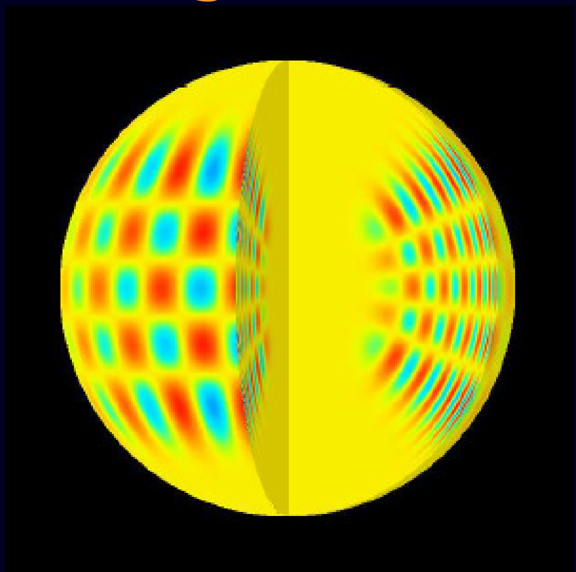
Eigenoscillations

- An eigenoscillation pattern of a string



$$v(x) = \sin(n+1)\pi x$$

- An eigenoscillation pattern of a sphere



$$v_{\mathrm{rad}}(r,\theta,\phi) = f_{nl}(r)Y_l^m(\theta,\phi)$$

n : the number of nodal surfaces

Helioseismology

- Solar eigenoscillation frequencies reflect interior structure of the sun

Solar structure → Eigenfrequencies

- In helioseismology, we try to reverse the path

Solar structure ← Eigenfrequencies

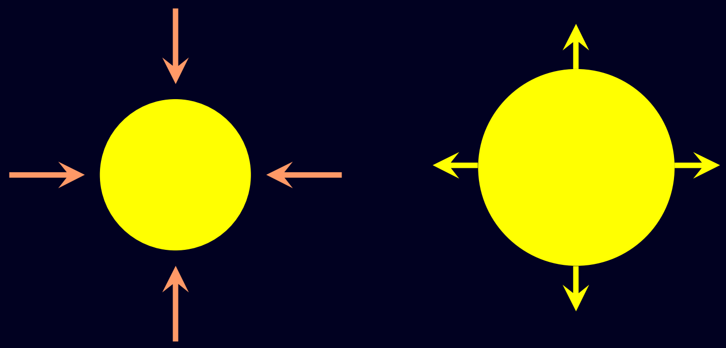
- This may be similar to…many things

What can helioseismology infer?

- A brief answer: whatever is determining the eigenfrequencies has a chance
- What determines the eigenfrequencies?
- That is to say, what kind of force is working on plasma that constitutes the sun?
  - Gas pressure
  - Gravity
    - Here we are neglecting rotation and magnetic fields

What can helioseismology infer?

- How gas pressure can work as a restoring force?
  - When a fluid element is squeezed, it bounces back (the strength is measured by 'bulk modulus')
  - This is sound wave.



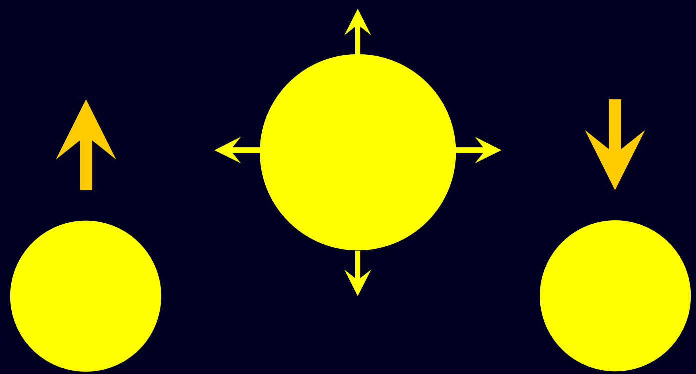
$$\left(\frac{\partial P}{\partial \ln \rho}\right)_{\mathrm{ad}} = \rho\left(\frac{\partial P}{\partial \rho}\right)_{\mathrm{ad}} = \rho c^2$$

P : pressure, ρ : density

c : adiabatic soundspeed

What can helioseismology infer?

- How gravity can work as a restoring force?
  - When a fluid element is pushed upwards it will expand
  - If the density afterward is larger than the surrounding, it sinks back to the original level
  - This is buoyancy oscillation.



$$\Delta\rho \cdot g = \left[\left(\frac{\partial \rho}{\partial P} \right)_{\mathrm{ad}} \frac{dP}{dr} - \frac{d\rho}{dr} \right] g \Delta r$$

$$\rho = \rho(r)$$

What can helioseismology infer?

- In short, what determine the solar eigenfrequencies, and thus can be inferred from helioseismology, are

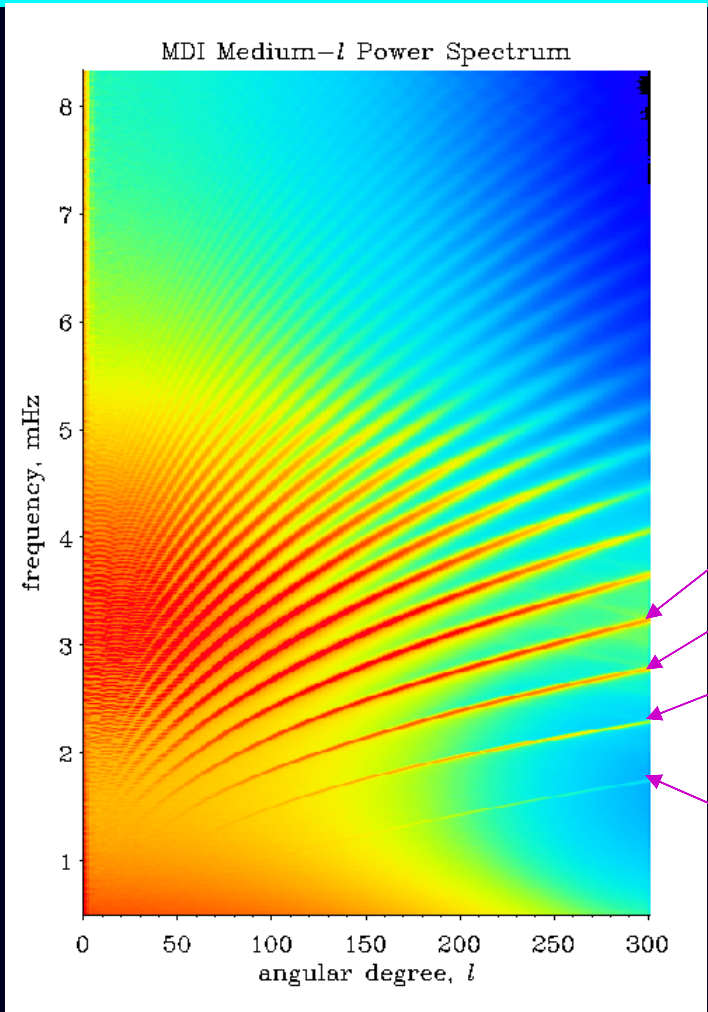
$c(r)$: adiabatic soundspeed distribution

$\rho(r)$: density distribution

Cowling's classification

- At high frequencies, pressure force dominates
  - Pressure modes: p modes
- At low frequencies, buoyancy force dominates
  - Gravity modes: g modes
- In between, there are incompressible surface-wave modes
  - Fundamental modes: f modes
    - Like ripples on the surface of a lake

The k–ω diagram



p modes, n=3

p modes, n=2

p modes, n=1

f modes

no g modes yet

Acoustic modes

- If the following condition is satisfied, the wave is almost purely acoustic

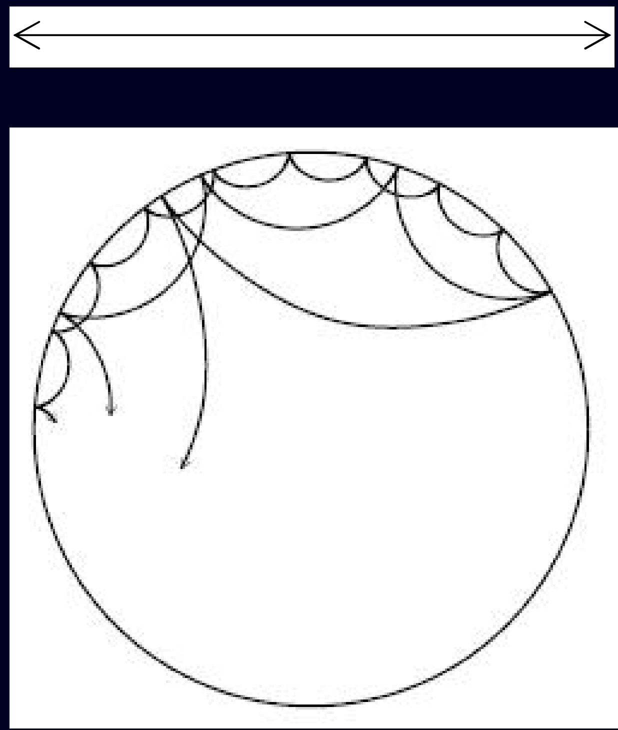
$$\nu >> \frac{1}{\tau_{\mathrm{dyn}}} \approx \mathrm{mHz}$$

$$\tau_{\mathrm{dyn}} = \sqrt{\frac{R^3}{GM}} \approx 1600\ \mathrm{s}$$

- The 5-minute oscillations are acoustic p modes

Ray theory

- At the high-frequency ('asymptotic') limit the propagation of sound wave in the sun can be well represented by a ray
- A ray path in the sun is not straight because of the variation in soundspeed



Fluid dynamical equation

- A more precise treatment requires perturbing fluid dynamic equations

$$\omega^2 \rho \vec{\xi} = -\nabla(\rho c^2 \nabla \cdot \vec{\xi}) - \nabla(\nabla P \cdot \vec{\xi}) + \frac{\nabla P}{\rho} \nabla \cdot (\rho \vec{\xi})$$
$$+ \rho \nabla \left[G \int \frac{\nabla' \cdot \left\{ \rho(\vec{r}\,') \vec{\xi}(\vec{r}\,') \right\}}{|\vec{r} - \vec{r}\,'|} dV' \right]$$

$\vec{\xi}(\vec{r})$: displacement vector
the fluid element at position $\vec{r}$
is now in position $\vec{r} + \vec{\xi}(\vec{r})$

the factor $e^{i\omega t}$ taken out

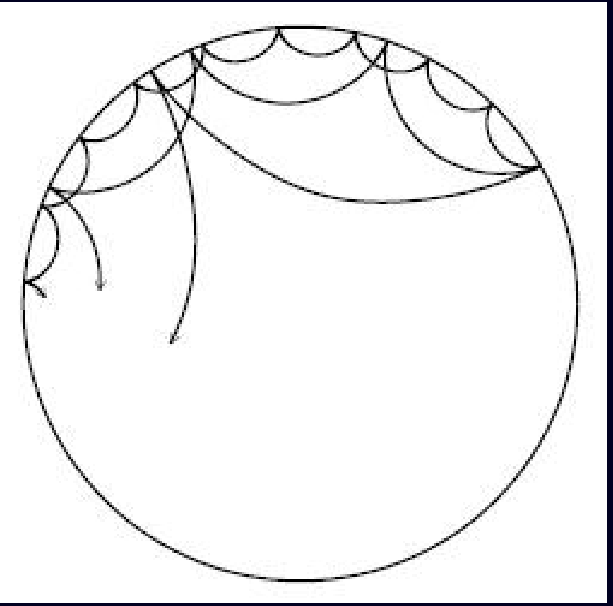
Ray theory vs. LAWE

- The (linear) perturbed fluid equation is often considered under adiabatic approximation
  - LAWE (Linear Adiabatic Wave Equation)
- Ray theory vs. LAWE
  - Ray theory is useful in revealing basic properties
  - LAWE is used for precise computations

Ray theory

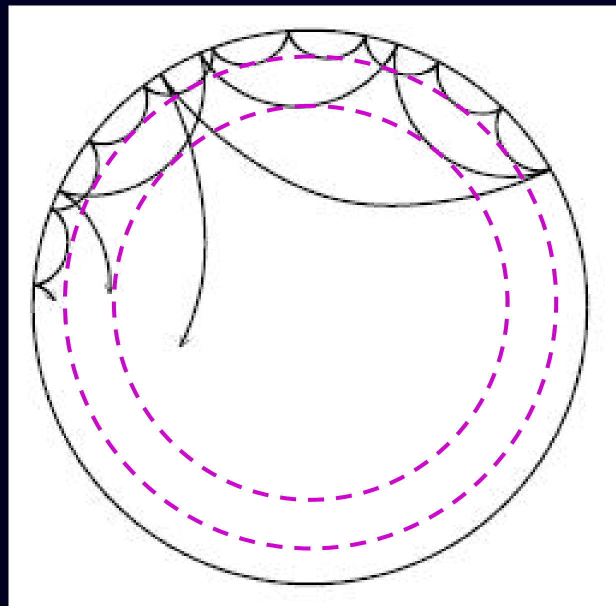
- Rays with steeper incident angles penetrate deeper into the solar interior
  - = smaller k_h (horizontal wave number)
  - = smaller ℓ

$$\omega = kc$$
$$k_h(R) = l / R$$



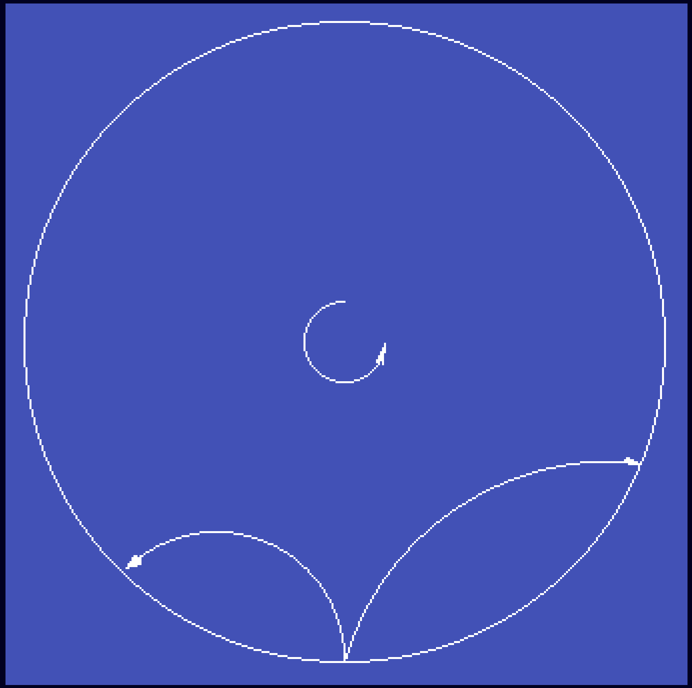
Ray theory

- Modes with smaller ℓ values penetrate deeper
- Different modes 'samples' different parts of the sun
- This is one reason why helioseismology works



Rotation of the sun

- Rotation of the sun affects the wave propagation
  - primarily by advection
  - also by Coriolis force

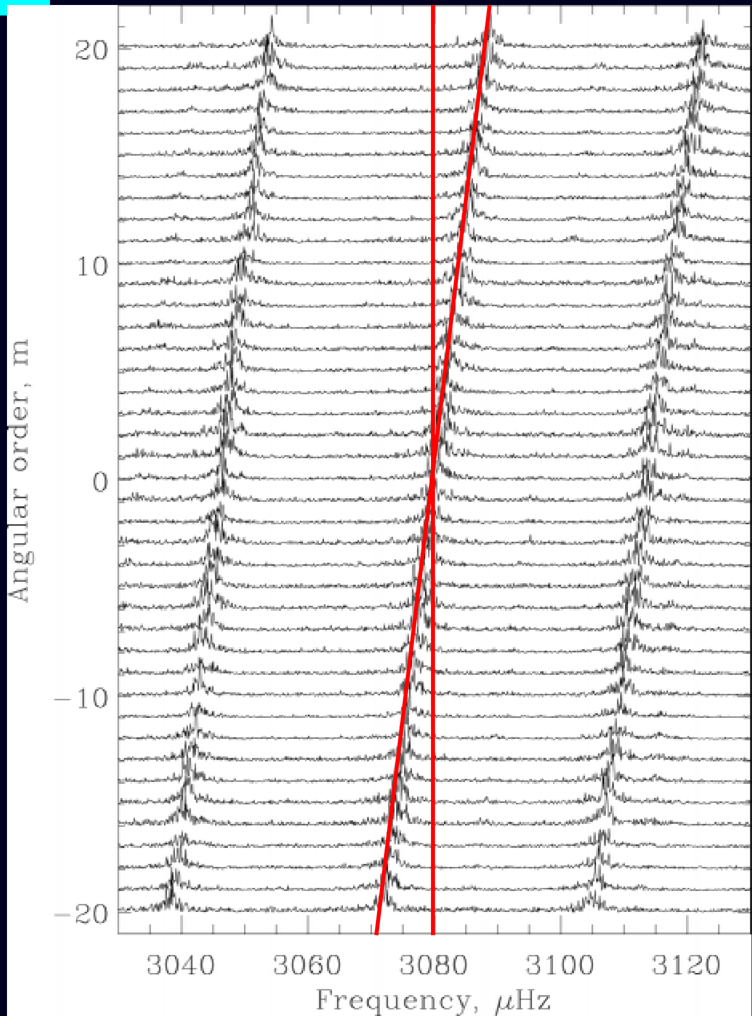


Rotational splitting

- As a result, the solar eigenfrequencies are shifted

$|a_{lm}(\omega)|^2$

the gradient ∝ rotational frequency



Rotational splitting

- By measuring rotationally split frequencies, we can infer the internal rotation of the sun

Seismic inferrence

- We can just listen to a musical instrument and tell what it is
  - Because we heard it before
- Nobody has listened to an object like the sun
  - We need physics and math to decode the sun's tone/timbre
    - forward problem: calculate the sun's tone from its structure
    - Inverse problem: infer the sun's structure from its tone

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Solar neutrino problem

- Neutrino: a byproduct of the thermonuclear reaction in the sun
- The amount of neutrino generated in the sun can be calculated from a solar model and can be used as a test of the model
- The number of neutrinos detected on earth is significantly smaller than the number expected

$$F_\nu[\text{observed}] \approx \left(\frac{1}{3} - \frac{1}{2}\right) \times F_\nu[\text{model}]$$

Solar neutrino problem

- Neutrino detection experiments
  - Davis's experiment
  - (Super) Kamiokande experiment
  - Gallium experiments (GALLEX, SAGE)
- All confirmed the deficit of neutrino flux
- Why fewer neutrinos?
  - Solar models are wrong?
  - Neutrino has some funny properties?

Neutrino Oscillations

- Neutrinos come in three flavours
  - Electron neutrino
  - Mu neutrino
  - Tau neutrino

$$\nu_e, \nu_\mu, \nu_\tau$$

- Neutrino oscillations: if neutrinos are NOT massless
  - they can change their flavours (and back)

- The old experiments detect (mostly) only electron-type neutrinos, hence the deficit

Solar neutrino problem solved?

- Sadbury Neutrino Observatory (SNO) experiment
  - Runs three kind of experiments

$$F_{\text{ES}} \approx \nu_e + \frac{1}{6}\left(\nu_\mu + \nu_\tau\right)$$

$$F_{\text{CC}} = \nu_e$$

$$F_{\text{NC}} = \nu_e + \nu_\mu + \nu_\tau$$

- Confirmed the following

$$F_{\text{NC}} = F_\nu[\text{model}]$$

Solar neutrino problem solved?

- The SNO results show that
  - the neutrino oscillations explanation is quantitatively consistent with the observation
- Most people think that the solar neutrino problem has thus been solved

Dynamical structure of the sun

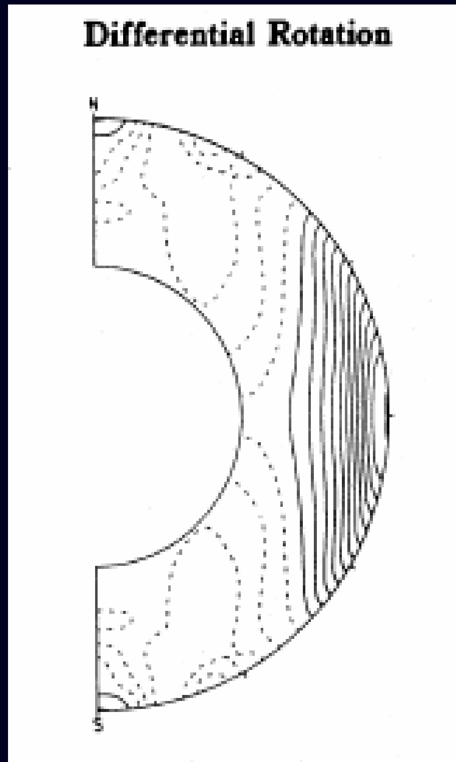
- Solar activity cycle is caused by a dynamo mechanism
- At the centre of any dynamo theory is interaction between flow and magnetic field
- How does the sun rotate?
- Cannot we just compute it?

Dynamical structure of the sun

- Solar convection zone is turbulent
  - Reynolds number ~ 10^{12}
  - Then Kolmogorov's scaling law states that the ratio between the largest scale and the smallest scale is ~ 10^{9}
    - This is the degree of freedom *per dimension*
  - To simulate the solar convection zone in 3D, one needs about 10^{27} grid points!

Dynamical structure of the sun

- One such (now old) attempt by Glatzmaier (1985)
  - 'Taylor columns' are seen
  - Taylor–Proudman's theorem
- Is this correct?
- Is there anyway to test this?
- Helioseismology can do that



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Inverse problem

- There is no agreed view on the definition of the 'inverse problem'
- In a broad view, any problem that is reversed is an inverse problem
  - If $x=3$, then $2x+1=7$. If $2x+1=7$, what is x? A. $x=3$
  - If $f(x)=x^2$, then $f'(x)=2x$. If $f'(x)=2x$, what is $f(x)$? A. $f(x)=x^2+C$

Inverse problem of solar structure

- Linear perturbation problem

structure $(\rho, c^2) \rightarrow$ eigenfrequencies $\{\omega_i\}$

perturbed structure $(\rho + \delta\rho, c^2 + \delta c^2)$
$\rightarrow$ eigenfrequencies $\{\omega_i + \delta\omega_i\}$

- Linear perturbation theory

$$\frac{\delta\omega_{nl}}{\omega_{nl}} = \int \left[K^{(nl)}_{c^2,\rho}(r) \frac{\delta c^2(r)}{c^2(r)} + K^{(nl)}_{\rho,c^2}(r) \frac{\delta\rho(r)}{\rho(r)} \right] dr$$

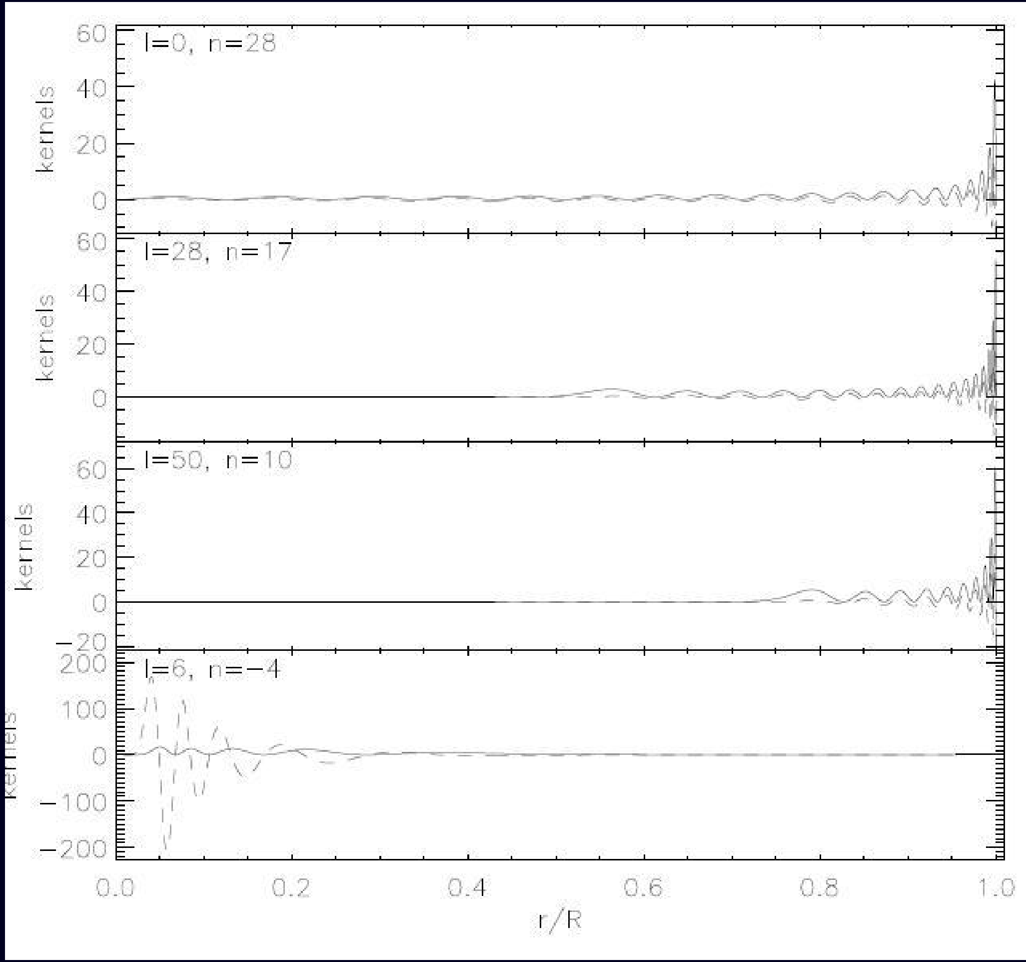
$$\delta q = q(\mathrm{sun}) - q(\mathrm{model})$$

Structure inversion kernels

- Examples

solid: $K_{c^2,\rho}^{(nl)}(r)$

dotted: $K_{\rho,c^2}^{(nl)}(r)$



Inverse problem of solar rotation

- Linear perturbation problem

the non-rotating sun $\rightarrow$ eigenfrequencies $\{\omega_i\}$

rotation $\Omega(r,\theta)$ as perturbation

$\rightarrow$ rotationally split eigenfrequencies $\{\omega_i + \delta\omega_i\}$

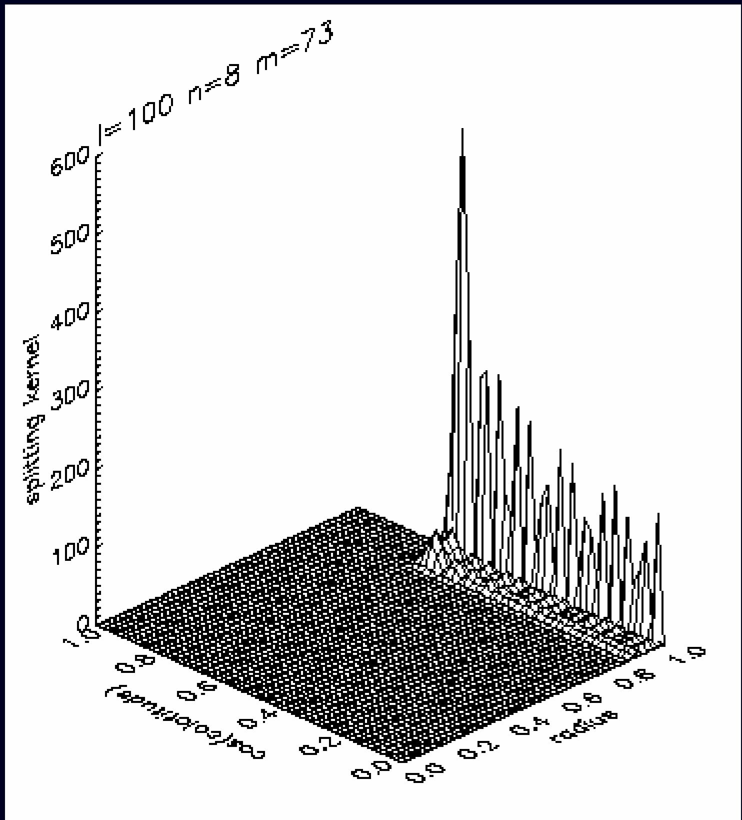
- Linear perturbation theory

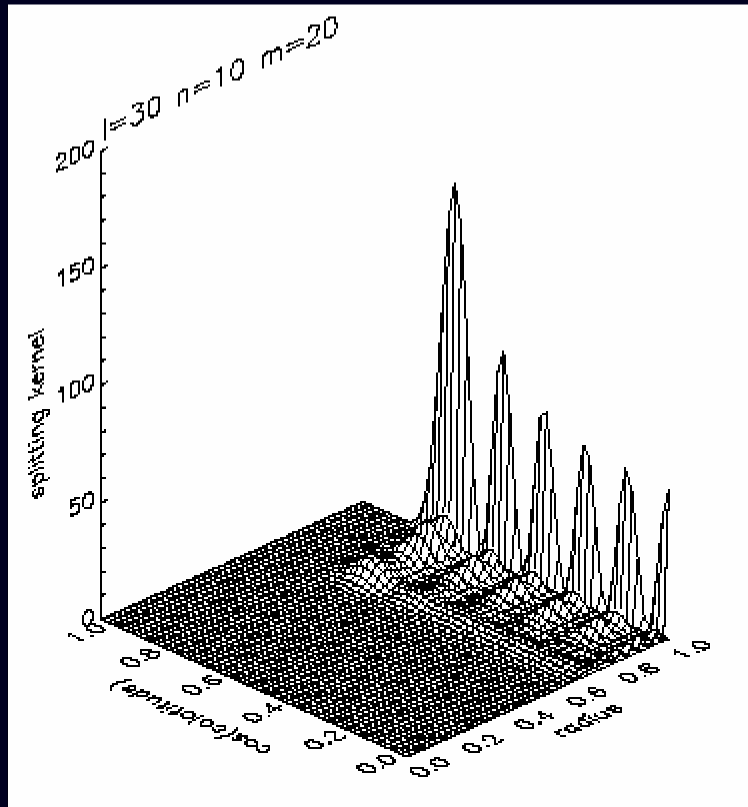
$$\delta\omega_{nlm} = \int K_{nlm}(r,\theta)\Omega(r,\theta)drd\theta$$

$$\delta q = q(\text{sun}) - q(\text{presumed non-rotating sun})$$

Rotation inversion kernels

- Examples of rotation inversion kernels





A canonical inverse problem

- Let us consider 1-d one-quantity inverse problem

$$b_i = \int K_i(x) f(x) dx + e_i \quad (i = 1, \ldots, M)$$

$f(x)$: what we are looking for

b_i : measurement i

$K_i(x)$: kernel function for measurement i

e_i : measurement error

  - The structure inversion is 1-d two-quantity
  - The rotation inversion 2-d one-quantity

A canonical inverse problem

- How do we 'solve' this?

$$b_i = \int K_i(x) f(x) dx + e_i \ (i = 1, \ldots, M)$$

  - A better wording would be: 'How do we estimate f(x) from the constraints?'
- There are ways, most of which involves *linear* operation
  - This amounts to make a linear combination of data for an estimate at the target position x_1

$$\hat{f}(x_1) = \sum_i c_i(x_1) b_i$$

A canonical inverse problem

- The estimate is related to f(x) in the following way

$$\hat{f}(x_1) = \sum_i c_i(x_1) b_i = \int \left[\sum_i c_i(x_1) K_i(x) \right] f(x)\,dx + \sum_i c_i(x_1) e_i$$

- With the additional definitions

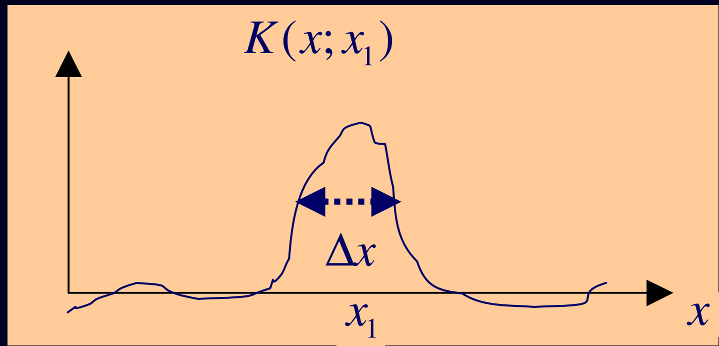
$$K(x; x_1) = \sum_i c_i(x_1) K_i(x) \quad \text{(averaging kernel)}$$

$$\delta\hat{f}(x_1) = \sum_i c_i(x_1) e_i$$

A canonical inverse problem

- It now looks like this:
  - The averaging kernel $K(x;x_1)$ gives us the spatial resolution of the estimate
  - The last term gives the statistical uncertainty of the estimate

$$\hat{f}(x_1) = \int K(x;x_1) f(x)\,dx + \delta\hat{f}(x_1)$$



$$\left\langle |\delta\hat{f}(x_1)|^2 \right\rangle = \sum_{ij} c_i(x_1) c_j(x_1) \left\langle e_i e_j \right\rangle$$

A canonical inverse problem

- How do we actually 'solve' them?
- Method 1: Optimally localized averaging
  - Localize $K(x;x_1)$ while keeping $\left\langle |\delta\hat{f}(x_1)|^2 \right\rangle$ low
- Method 2: Regularized Least-Squares
  - Discretize the problem

$$b = Kf \quad (b: M\times 1,\ K: M\times N,\ f: N\times 1)$$

  - Then solve the regularized equation

$$(K^T K + \alpha L^T L) f = K^T b$$

L: a smoothing matrix such as a discretized expression of $\frac{d^2}{dx^2}$

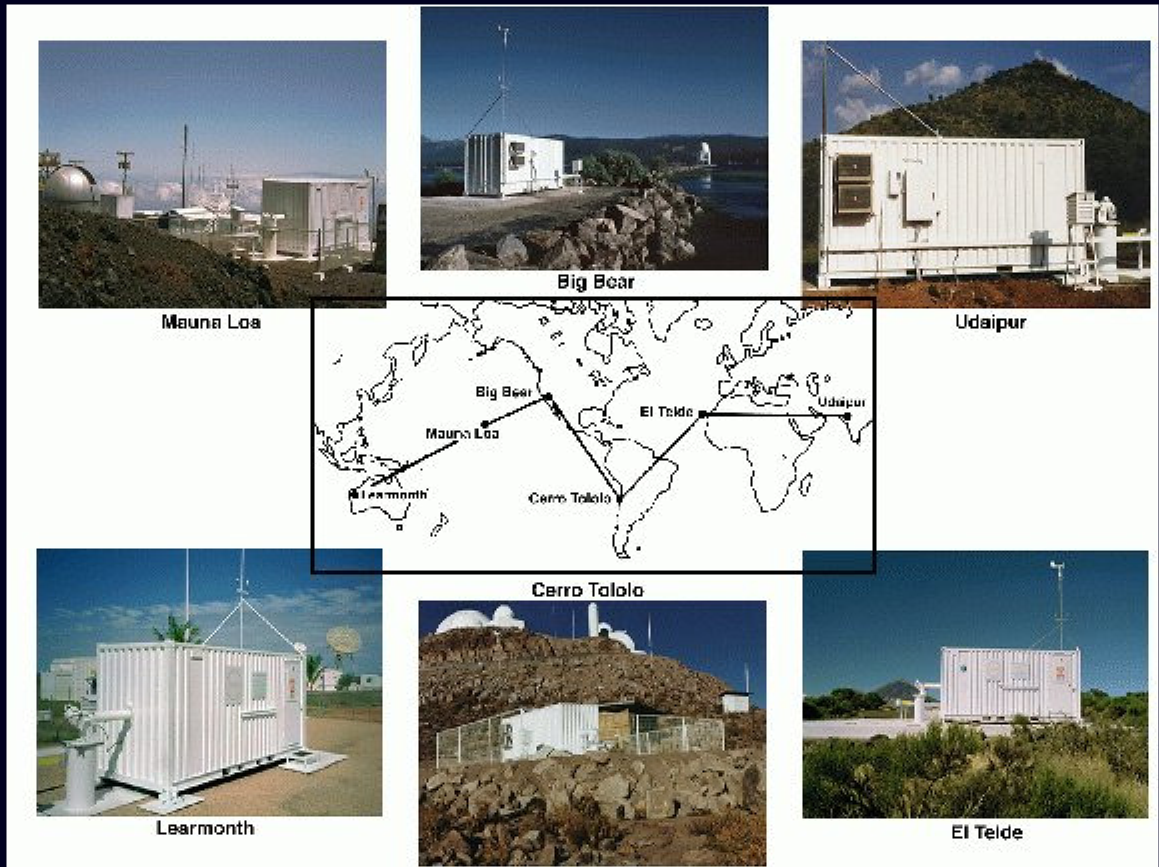
α: regularization parameter

Helioseismic obervation

- Precise measurement of eigenfrequencies
  - Needs a long continuous observations
- However, the sun sinks below the horizon at night

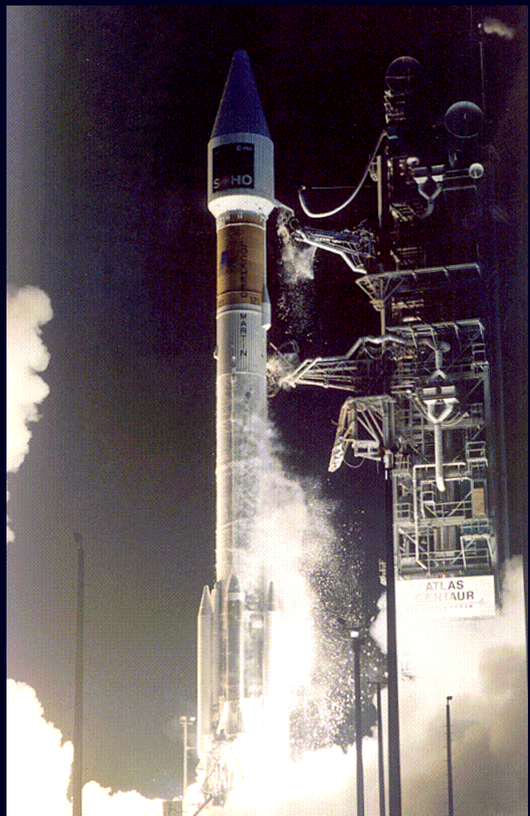
GONG project

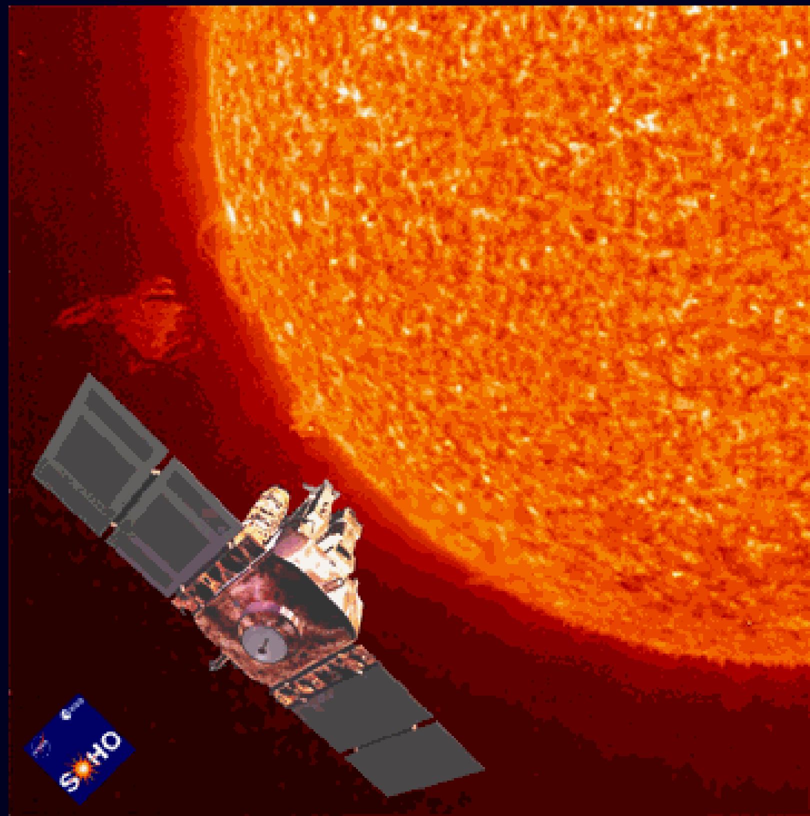
- A ground-based network



SOHO spacecraft

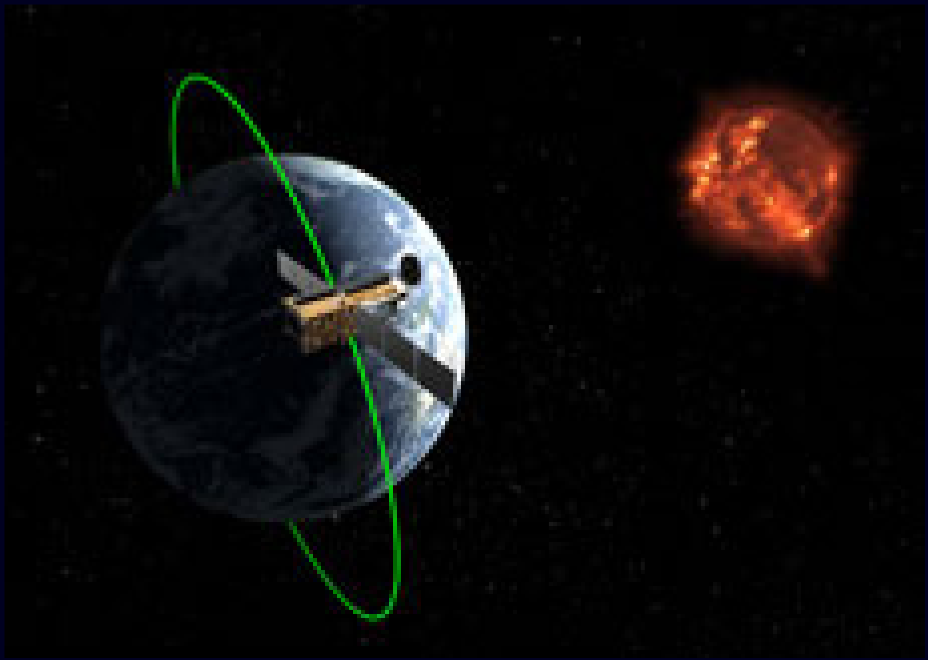
- Stationed at L_1 point





Hinode satellite

- In a sun-synchronous orbit

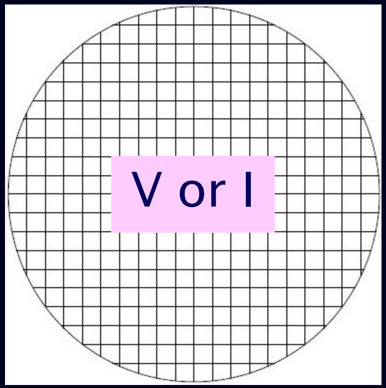


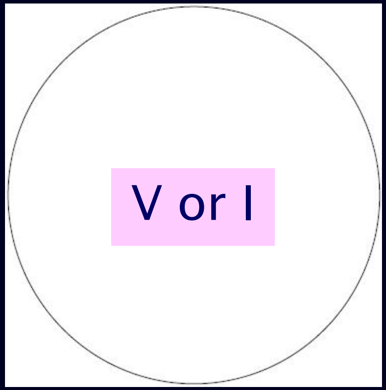
Helioseismic observation

- Velocity or intensity
  - Doppler velocity measurement exhibits higher S/N ratio
  - Except near the limb, where the Doppler signal weakens due to projection
- Resolved or full-disc
  - We need resolved observations for spherical-harmonic decomposition
  - Except low-degree modes where CCD's tempo-spatial stability matters

Helioseismic obervation

- It helps to have variety of combinations
  - MDI (onboard SOHO): R, V&I
  - GONG: R, V
  - TON: R, I
  - BiSON: F, V
  - SPM (on board SOHO): F, I
  - SOT (onboard Hinode): R, V&I
  - …and a lot more

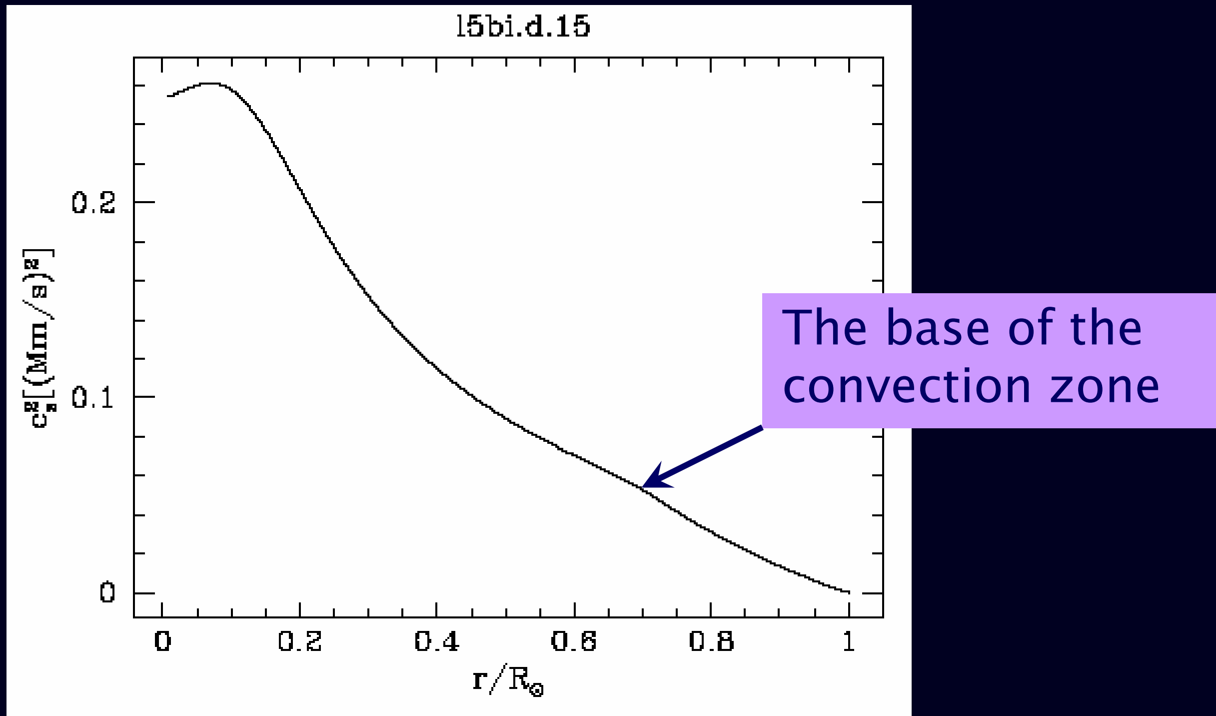




Soundspeed distribution in the sun

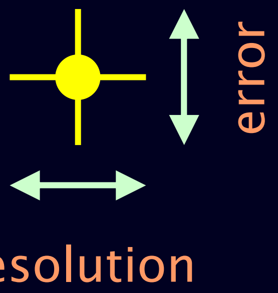
- According to a standard solar model

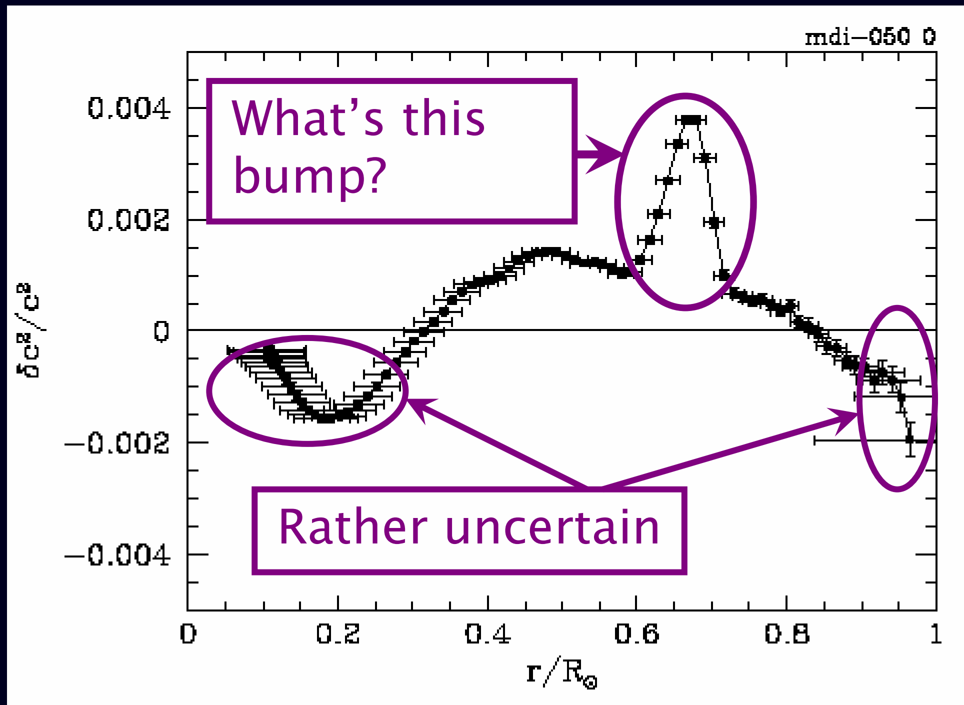
$$c^2 \propto \frac{T}{\mu}$$



Soundspeed inversion

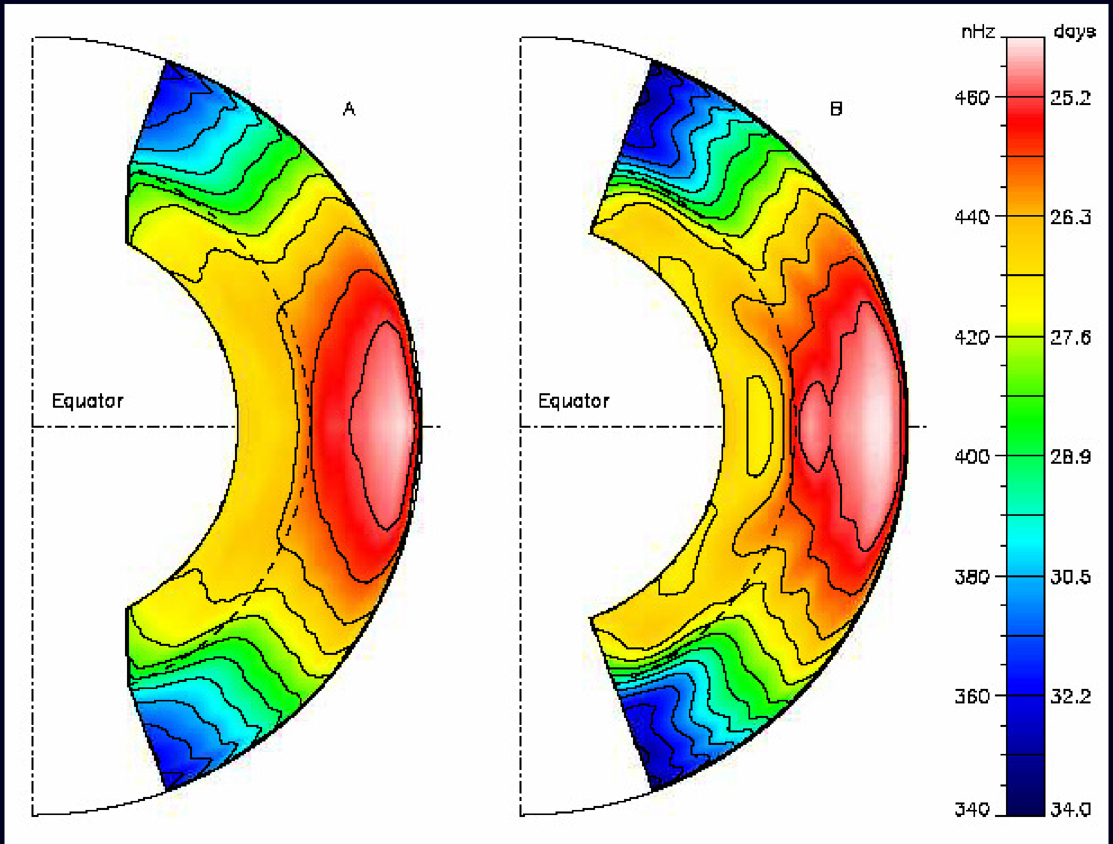
- This modern model agrees with the 'observation' within a half per cent accuracy

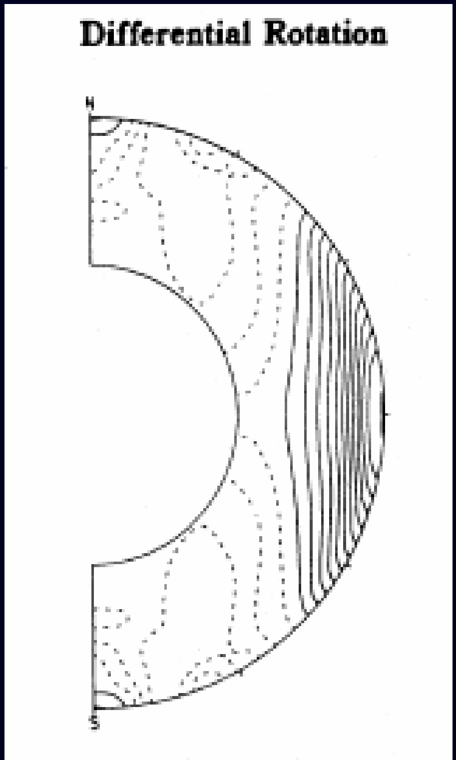




Rotation inversion

- Solar differential rotation





Glatzmaier (1985)

Rotation inversion

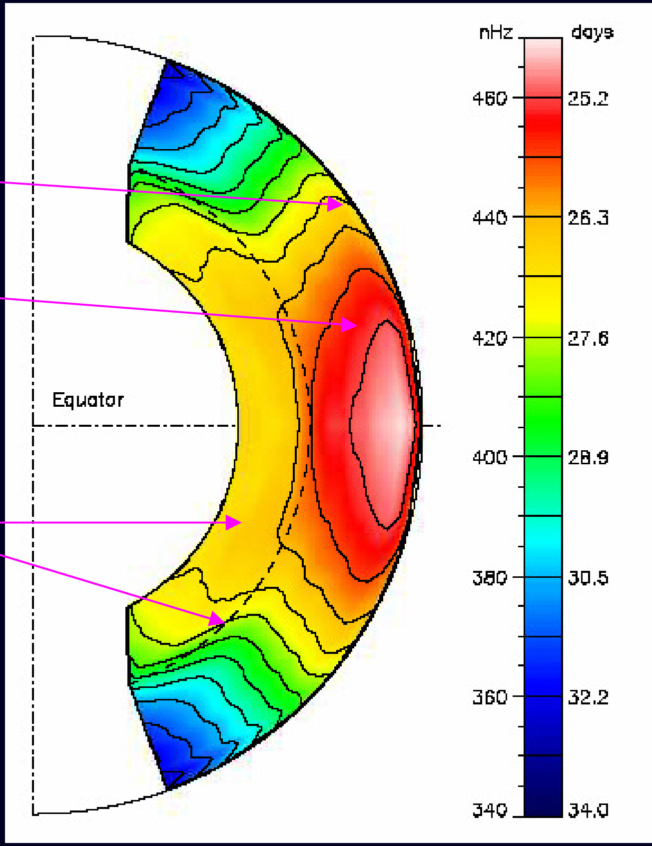
- Solar differential rotation

Surface shear layer

Non-columnar rotation

'Tachocline'

Rigid rotation?

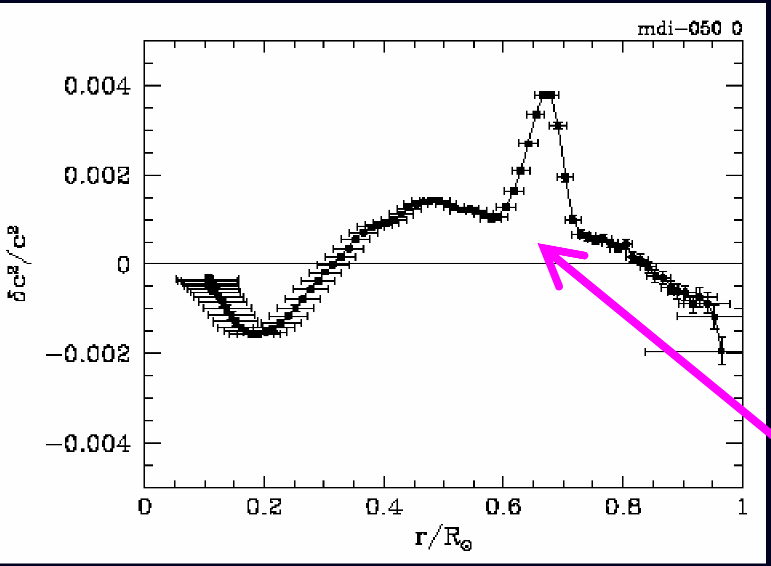


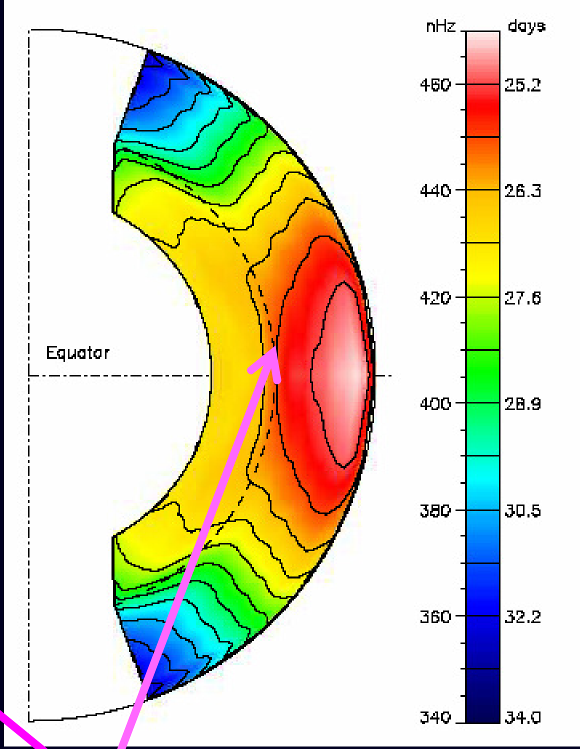
Previous dynamo calculations all rejected

Tachocline

- A steep gradient in the rotation rate

Dynamo action?





Shear-induced mixing counteracting gravitational settling of Helium?

Tachocline

Gravitational settling

- Heavier elements sink i.e. migrate towards deeper layers
  - In reality what happens is lighter elements diffuse upwards faster than heavier elements
  - Modern standard models take account of Helium accumulation beneath the base of the convection zone
- If the accumulation is overestimated, then
  - mean molecular weight overestimated
  - soundspeed underestimated

Global-mode inversions

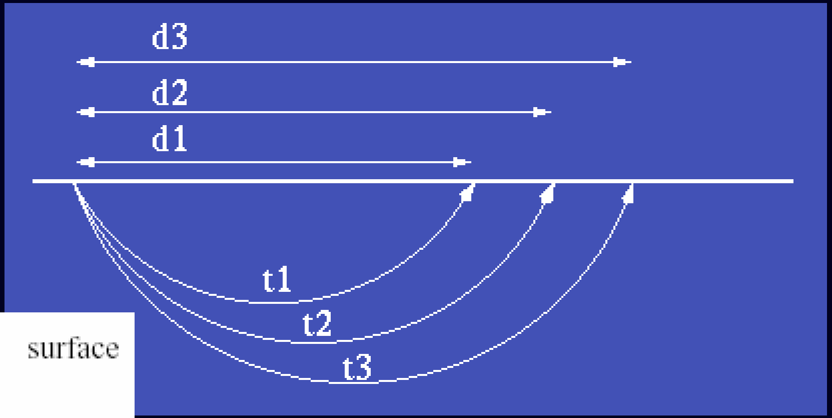
- Have revealed that
  - Modern standard models are very good except
    - near the base of the SCZ: extra mixing?
    - near the surface
    - the central region
  - The sun rotates rather differently from previously thought
    - tachocline: a seat to many dynamical processes such as mixing and dynamo?
    - small-scale dynamo may be sustained by the surface shear layer?

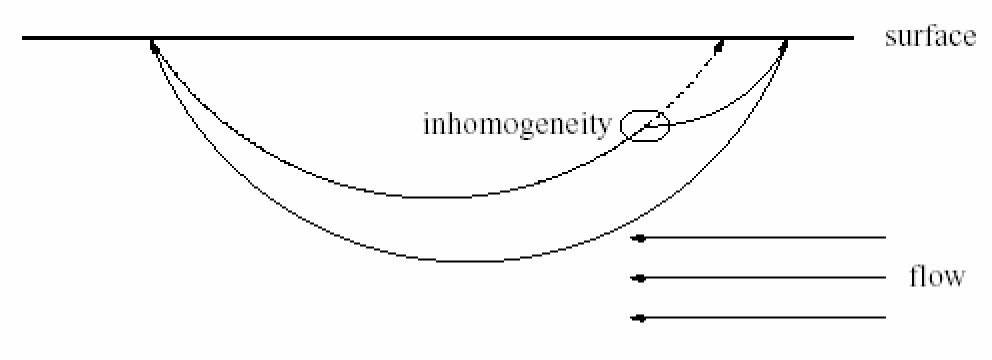
Today’s lecture

- The sun is oscillating
- How we can see through the photosphere of the sun
- What are the issues with the sun anyway?
- Inversion of Global-mode frequencies and main results
- Development of Local helioseismology

Local helioseismology

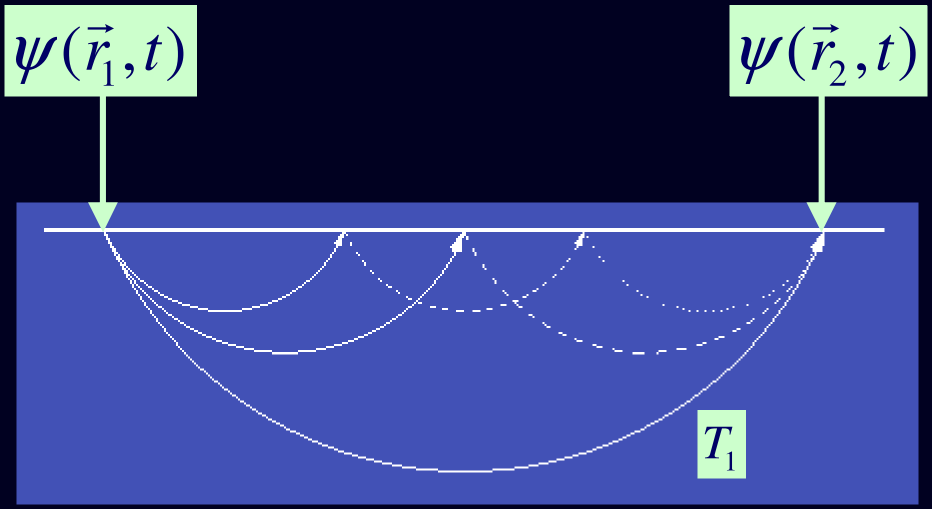
- Forget the global modes
- Direct measurement of subsurface propagation of waves





Time-distance method

- Cross-correlation function

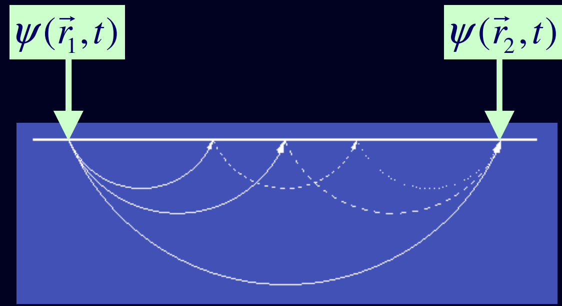


$$C(\vec{r}_1, \vec{r}_2, \tau) = \int \psi^*(\vec{r}_1, t)\psi(\vec{r}_2, t+\tau)dt$$

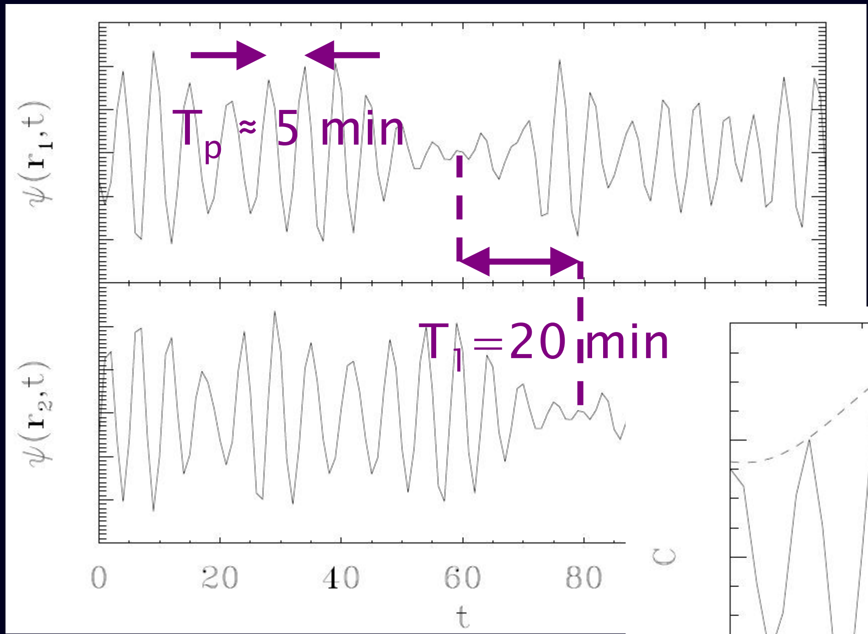
C is large around $\tau \approx T_1$

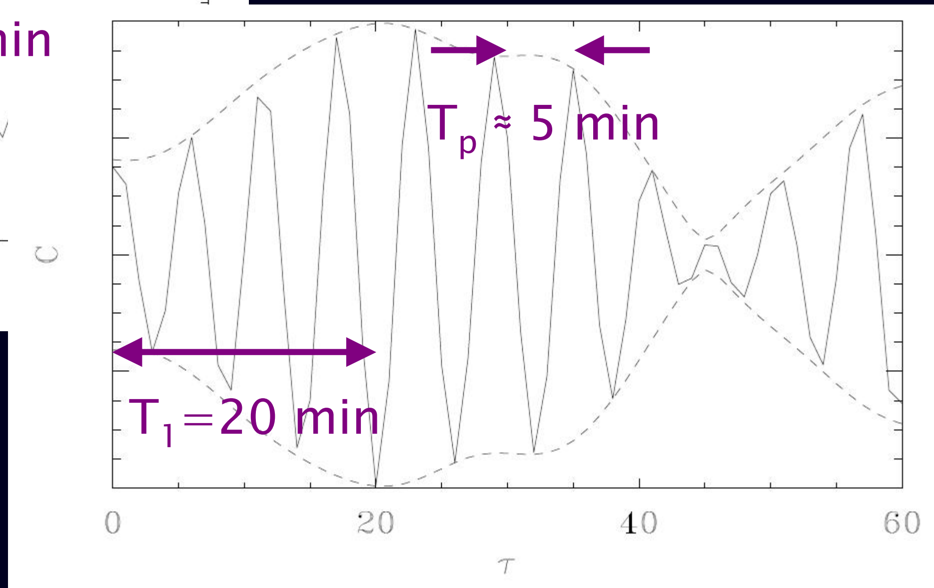
Cross correlation function

- An (artificial) example



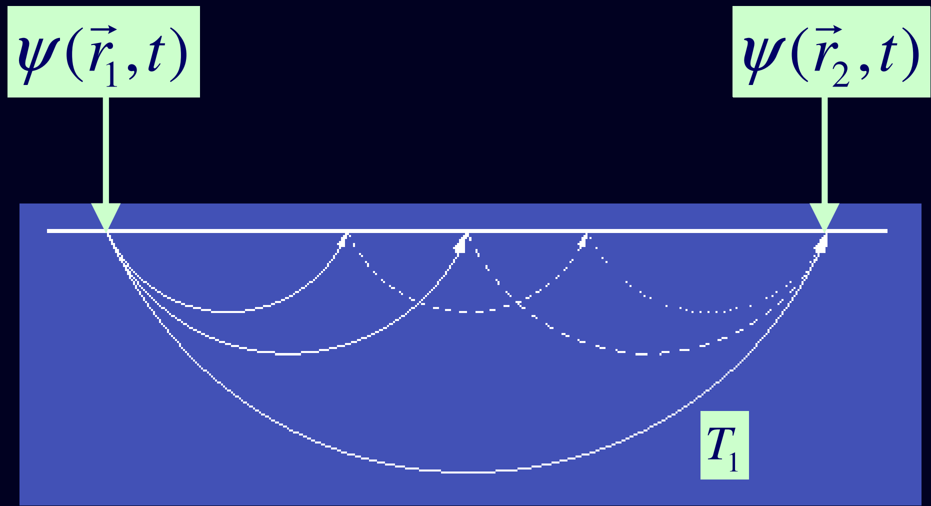
$$C(\vec{r}_1,\vec{r}_2,\tau)=\int\psi^*(\vec{r}_1,t)\psi(\vec{r}_2,t+\tau)dt$$





Time-distance method

- Cross-correlation function



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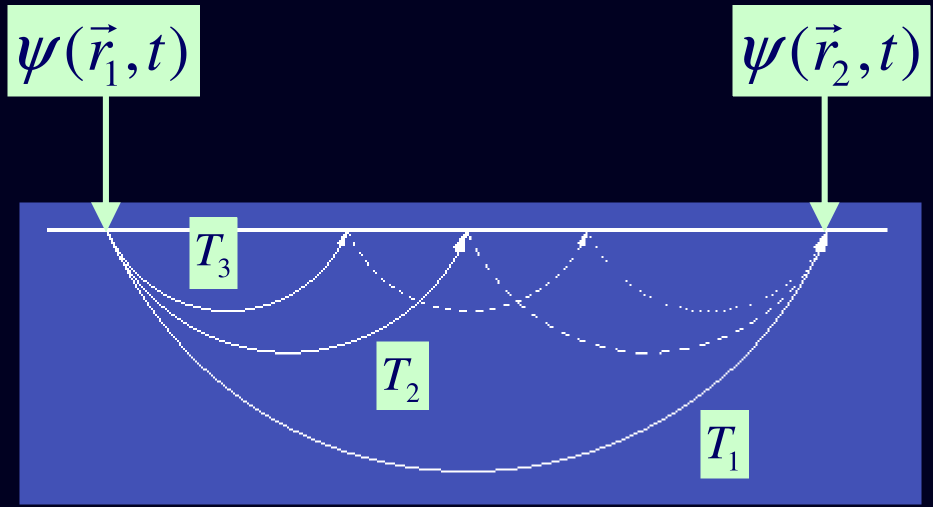
Time-distance method

- Cross-correlation function

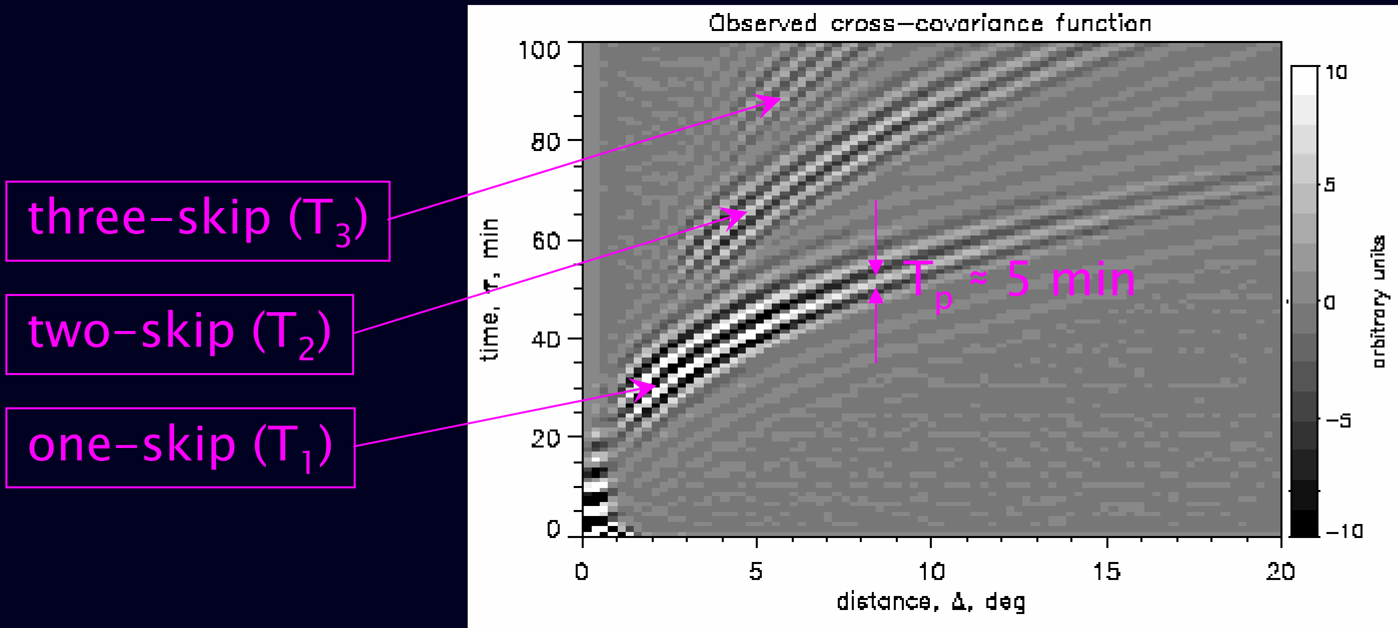


$$C(\vec{r}_1,\vec{r}_2,\tau)=\int \psi^*(\vec{r}_1,t)\psi(\vec{r}_2,t+\tau)dt$$

C is large around $\tau \approx T_1,\ 2T_2,\ 3T_3$

Time–distance method

- A solar time–distance diagram

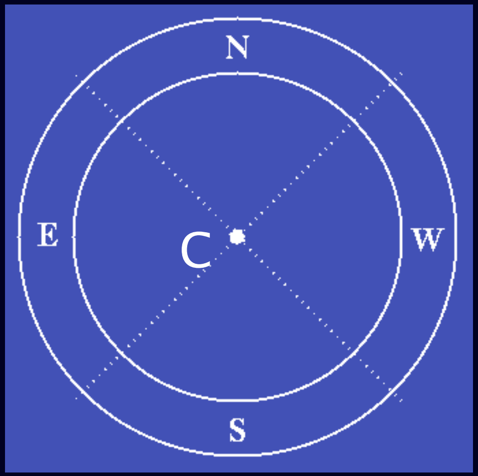


Cross–correlation function

$$C(\Delta_2,\tau)=\int_{|\vec{r}_1-\vec{r}_2|=\Delta}\psi^*(\vec{r}_1,t)\psi(\vec{r}_2,t+\tau)d\vec{r}_1 d\vec{r}_2 dt$$

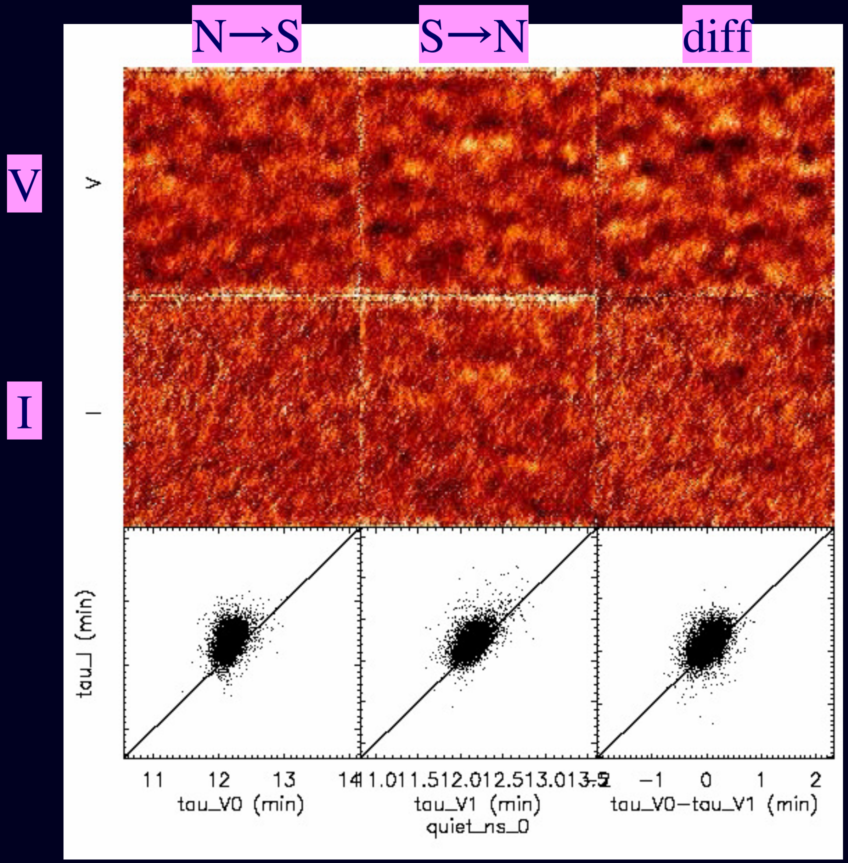
Travel-time map

- For a real localized inference, we use travel time map
  - Around a target (C) are set up annuli and segments on them (NEWS, in this example)
  - Calculate cross-correlations (and hence travel times) between
    - C-NEWS (I/O), NEWS-C (O/I)
    - EW, WE, NS, SN
    - etc



Travel-time maps

- Quiet, small annulus(0.306-0.714 deg)



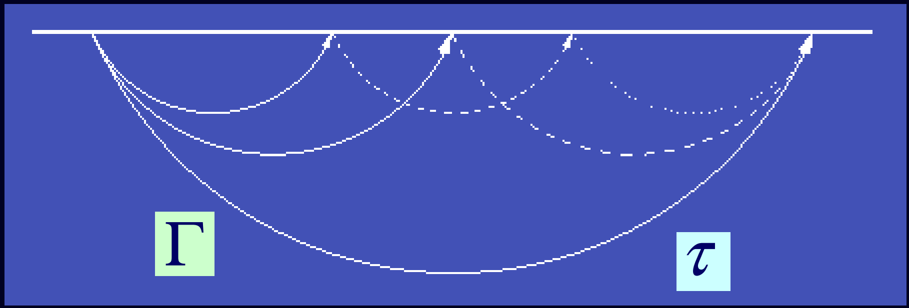
Travel–time perturbation

- Using ray approximation

$$\tau \approx \int_\Gamma \frac{dl}{c} = \int_\Gamma \frac{\vec{k}\cdot d\vec{l}}{\omega}$$

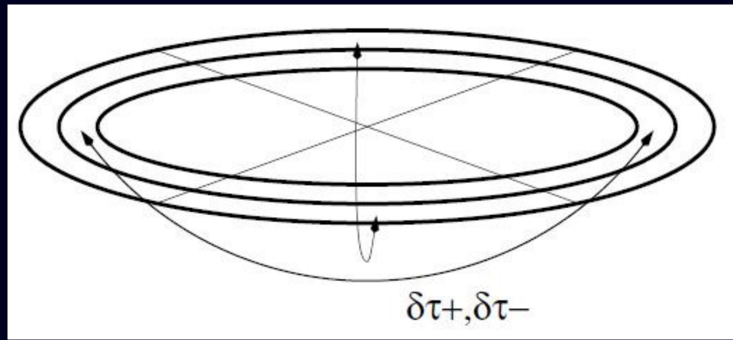
- Soundspeed perturbation δc and flow velocity $\vec{v}$ lead to

$$\delta\tau \approx \frac{1}{\omega}\int_\Gamma \delta\vec{k}\cdot d\vec{l} \approx -\int_\Gamma \frac{\delta c}{c^2}dl - \int_\Gamma \frac{\vec{v}\cdot d\vec{l}}{c^2}$$



Travel-time inversion

- Mean and differential travel-time perturbations



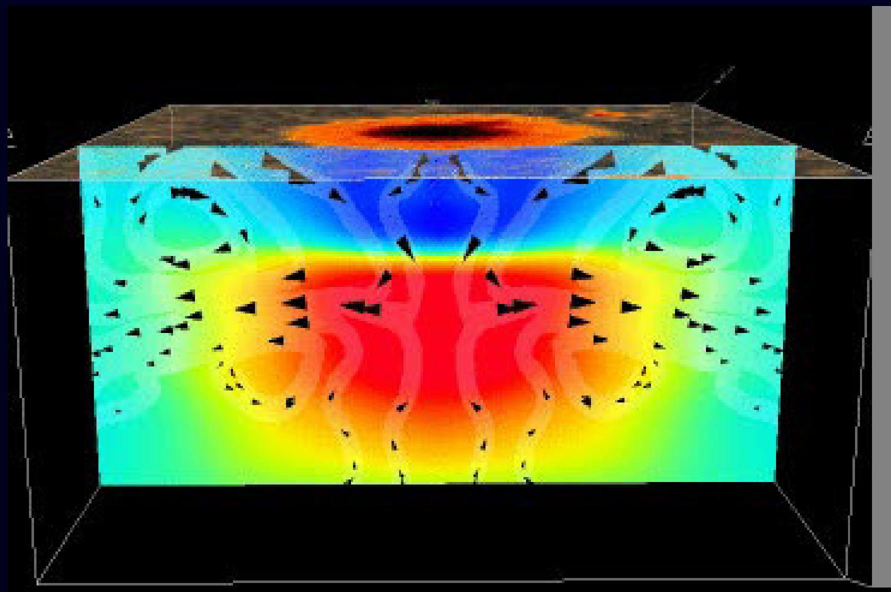
$$\frac{\delta\tau_+ + \delta\tau_-}{2} \approx -\int_\Gamma \frac{\delta c}{c^2} dl$$

$$\frac{\delta\tau_+ - \delta\tau_-}{2} \approx -\int_\Gamma \frac{\vec{v}\cdot d\vec{l}}{c^2}$$

- Bases for inversion analyses

Structure around a sunspot

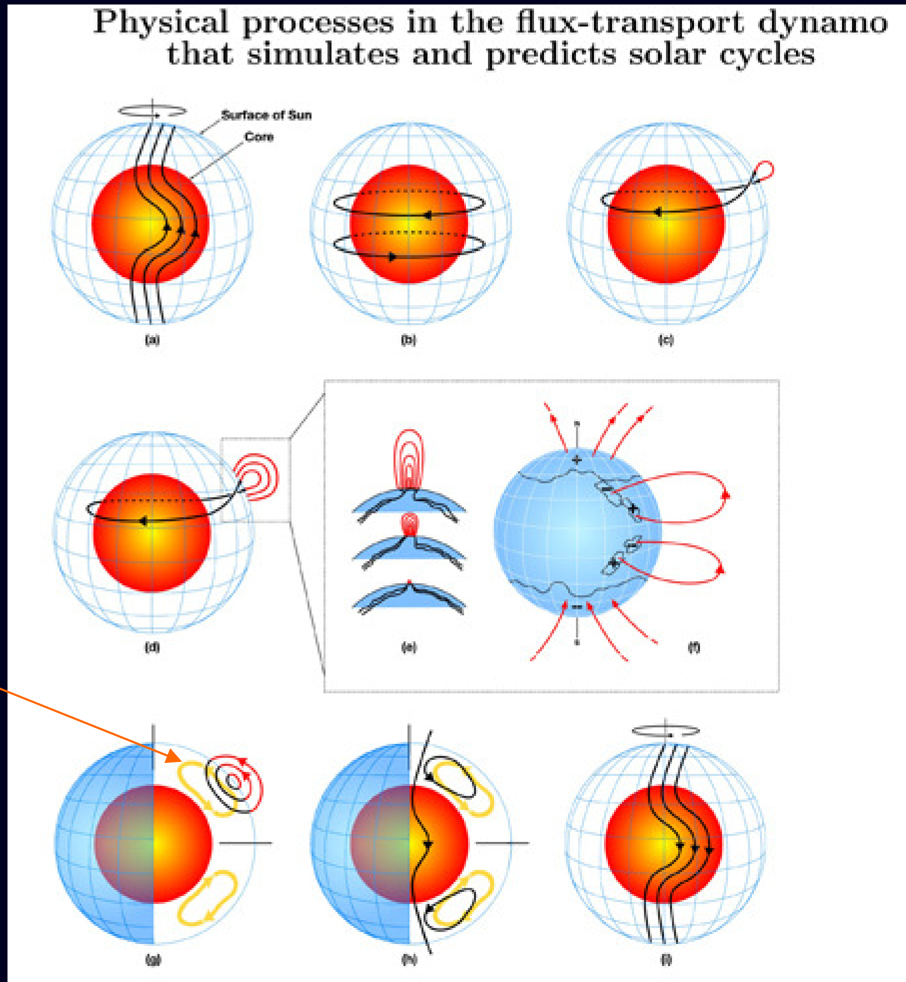
- From time-distance method



Parker's spaghetti model?

Meridional flow

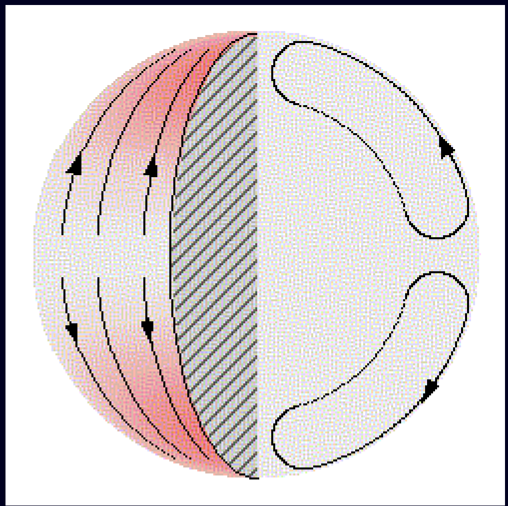
- An important ingredient in dynamo theory

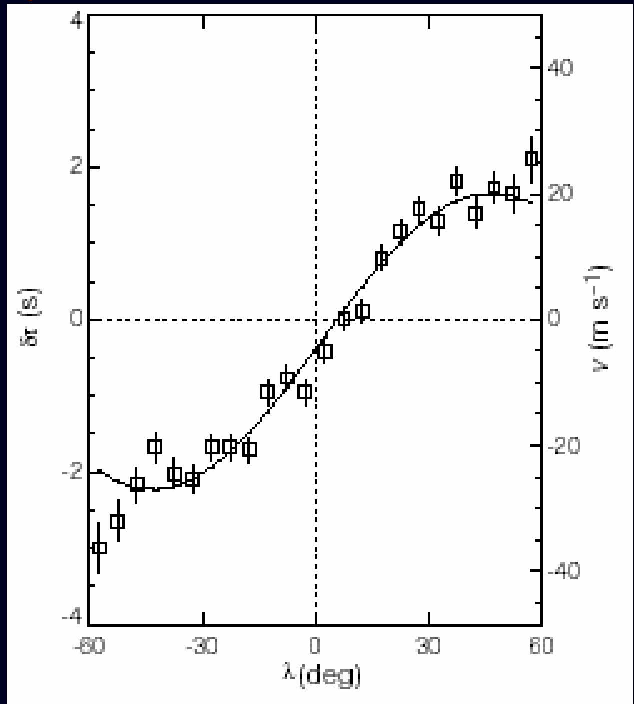


Meridional flow

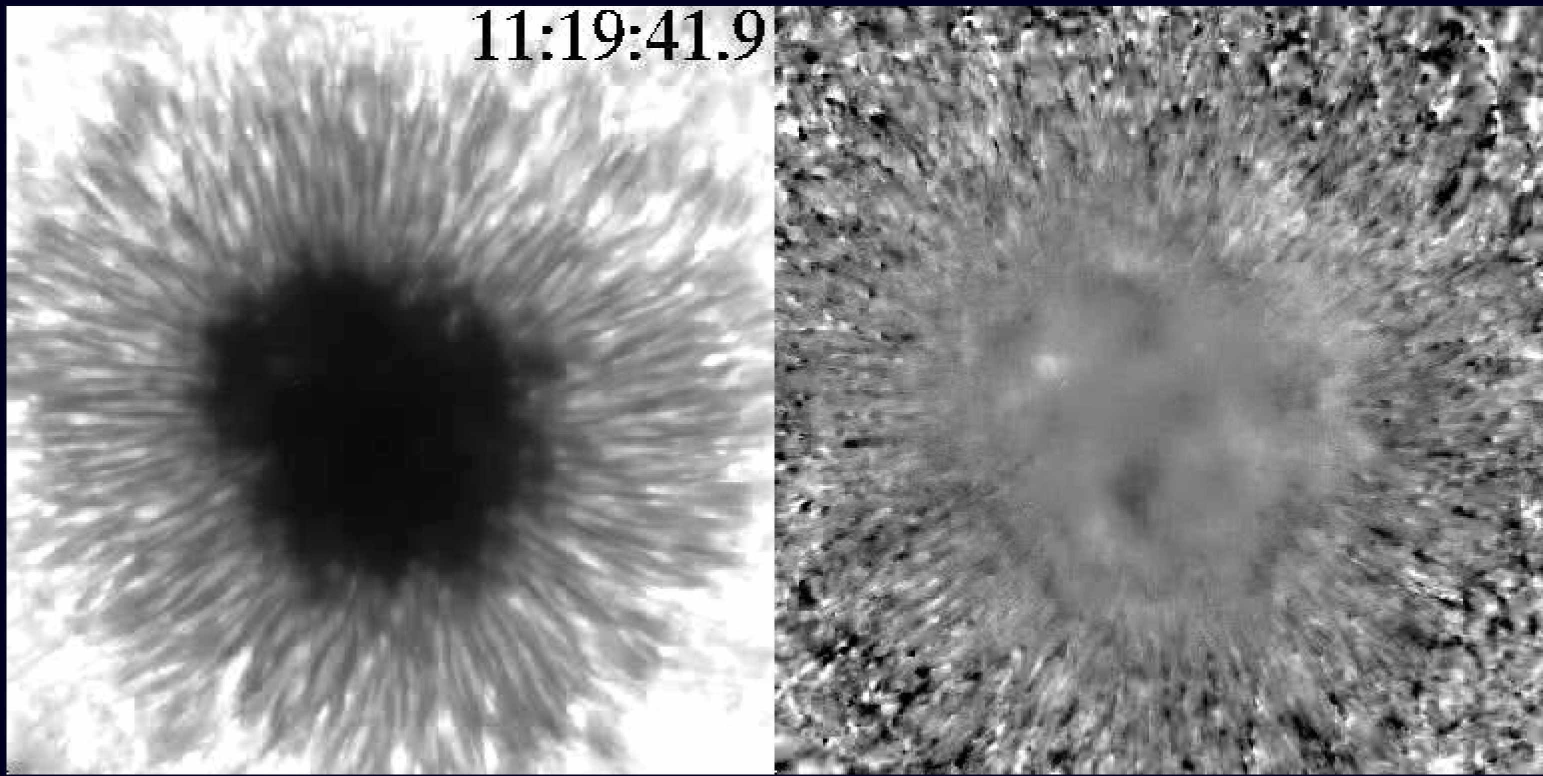
Meridional flow

- Time-distance measurement of meridional flow
  - Poleward, up to 20m/s





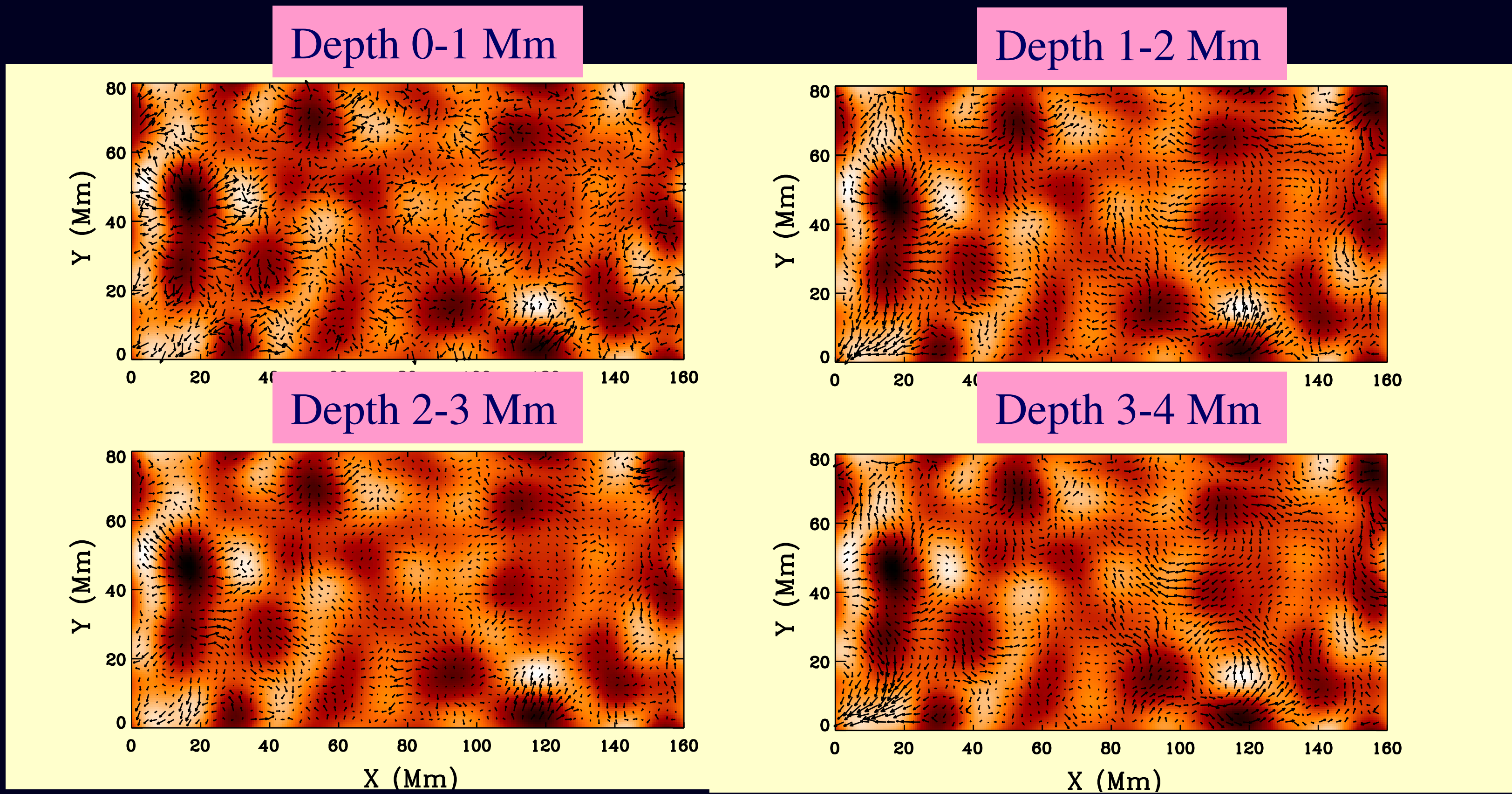
Umbral oscillations



Nagashima et al (2007)

A clue to investigate sunspot structure beneath and above the photosphere?

Supergranulation patterns

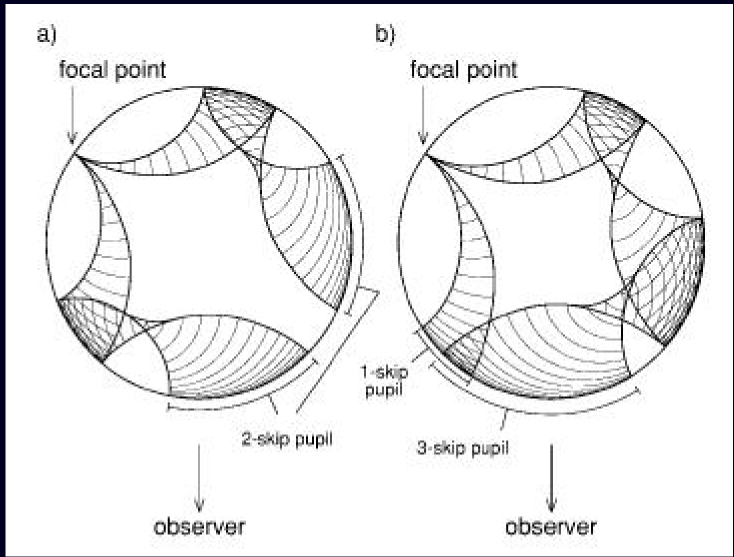


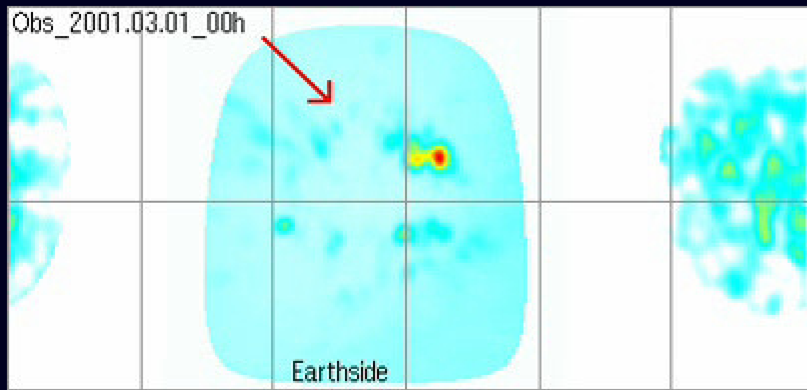
Sekii et al (2007)

Background: supergranulation pattern seen in O/I travel time difference

Far-side imaging

- The other side of the sun





Summary

- Helioseismology is about measuring physical quantities *inside* the sun, based on wave/oscillation phenomena on the solar surface
  - Global mode inversions have revealed the internal structure as well as the internal differential rotation
  - Local helioseismology, still an immature discipline, promises to tell us more about inhomogeneous static and dynamic structure of the sun